University of Calgary
Fall semester 2013

PHYS 615: Advanced Quantum Mechanics I

Final examination

December 12, 2013, 8:00 am

Open books. No electronic equipment allowed.
Full credit = 100 points. Attempt all problems. Partial credit will be given.

Problem 1 (10 pts). In the Schrödinger picture, the commutator of two operators $\hat{A}$ and $\hat{B}$ equals some third operator $\hat{C}$. Find the commutator of operators $\hat{A}(t)$ and $\hat{B}(t)$ in the Heisenberg and interaction pictures. The relevant Hamiltonians are known.

Problem 2 (25 pts). A superposition of the single-photon and vacuum states $\alpha|0\rangle + \beta|1\rangle$ is incident on a beam splitter with energy transmissivity T.

a) Find the two-mode state after the beam splitter.

b) Find the state in the transmitted channel if the reflected channel is discarded.

c) The reflected channel is subjected to a measurement by a photon detector with quantum efficiency η. Find the state in the transmitted channel in the events of a "click" and "no click", as well as corresponding probabilities.

Problem 3 (30 pts). A spin-1/2 particle, with the spin initially oriented along the z axis ($\psi_\uparrow(0) = 1, \psi_\downarrow(0) = 1$), evolves in a magnetic field whose vector is oriented between the positive x and z axis, at angle θ to the z axis. The interaction Hamiltonian is $\hat{H} = \mu\hat{\vec{\sigma}} \cdot \vec{B}$, where μ is the Bohr magneton.

a) Write the matrix of the Hamiltonian in the basis $(\{|\psi_\uparrow\rangle, |\psi_\downarrow\rangle\})$.

b) Find the state of the particle's spin at arbitrary moment t.

c) Sketch the trajectory of the Bloch vector in accordance with the Bloch vector precession equation. Using geometry, find the state corresponding to the lowest point of the trajectory. At what moment t is this point reached for the first time? Verify consistency of your answers with the result of part (b).

Problem 4 (35 pts). A two-level atom initially in the ground state is undergoing the following procedure.

- Unitary evolution with matrix $\hat{U} = (1/\sqrt{2}) \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$ in the $\{|e\rangle, |g\rangle\}$ basis.
- Interaction for time t with electromagnetic field in a cavity under interaction Hamiltonian $\hat{H}_I = \hbar G \hat{a}^\dagger \hat{a} \, |e\rangle\langle e|$. The field is initially in a coherent state $|\alpha\rangle$.
- Unitary evolution with matrix $\hat{U}$.

a) What is the generic name for this experimental procedure?

b) Find the state of the field-atom system after each step. The field state can be expressed in the photon number basis.

c) After the above procedure, the atom is measured in the $\{|e\rangle, |g\rangle\}$ basis. Find the interaction time t_0 such that this measurement is equivalent to a measurement of the parity operator of the initial field state.

d) Find the state of the field for each possible result of the above measurement.

e) Find the Hamiltonian $\hat{H}$ such that $\exp(-i\hat{H}t_1/\hbar)=\hat{U}$ for some t_1.