

Third mid term

Solutions

1 a) $|l=8, m=8\rangle = |m_1=5\rangle \otimes |m_2=3\rangle$ (homework)
 $\Rightarrow C-G = 1$

b) $\langle l=8, m=8 | \langle l_1=5, m_1=5 \rangle \otimes |l_2=3, m_2=3\rangle + |l_1=5, m_1=5 \rangle \otimes |l_2=3, m_2=3\rangle$
 $\sqrt{8 \cdot 9 - 8 \cdot 7} \langle l=8, m=7 | = \sqrt{5 \cdot 6 - 5 \cdot 4} |m_1=4, m_2=3\rangle$
 $+ \sqrt{3 \cdot 4 - 3 \cdot 2} |m_1=5, m_2=2\rangle$
 $\langle m_1=5, m_2=2 | l=8, m=7 \rangle = \frac{\sqrt{3 \cdot 4 - 3 \cdot 2}}{\sqrt{8 \cdot 9 - 8 \cdot 7}} = \frac{\sqrt{6}}{4}$

2 a) $\langle + | \Psi \rangle = \frac{1}{3\sqrt{2}} (\langle H | + \langle V |) (|HH\rangle + 2|HV\rangle - 2|VV\rangle)$
 $= \frac{1}{3\sqrt{2}} (|H\rangle + 2|V\rangle - 2|V\rangle) = \frac{1}{3\sqrt{2}} |H\rangle$ $pr = \frac{1}{18}$

$\langle - | \Psi \rangle = \frac{1}{3\sqrt{2}} (\langle H | - \langle V |) (|HH\rangle + 2|HV\rangle - 2|VV\rangle)$
 $= \frac{1}{3\sqrt{2}} (|H\rangle + 2|V\rangle + 2|V\rangle) = \frac{1}{3\sqrt{2}} (|H\rangle + 4|V\rangle)$ $pr = \frac{17}{18}$

b) $P = \frac{1}{18} (|H\rangle \langle H| + (|H\rangle + 4|V\rangle)(\langle H| + 4\langle V|)) = \frac{1}{18} \begin{pmatrix} 2 & 4 \\ 4 & 16 \end{pmatrix} = \frac{1}{9} \begin{pmatrix} 1 & 2 \\ 2 & 8 \end{pmatrix}$

c) $\text{Tr}_A |\Psi\rangle \langle \Psi| = \langle H | \Psi \rangle \langle \Psi | H \rangle + \langle V | \Psi \rangle \langle \Psi | V \rangle$
 $= \frac{1}{9} [(|H\rangle + 2|V\rangle)(\langle H| + 2\langle V|) + 4|V\rangle \langle V|] = \frac{1}{9} \begin{pmatrix} 1 & 2 \\ 2 & 8 \end{pmatrix}$

3 Echo amplitude is proportional to the horizontal component of Bloch vector $\Rightarrow \propto \sin \theta$

