PHYS 543: Quantum Mechanics II

Second midterm examination

October 20, 2017, 10:00–10:50, ST 061

Problem 1. (40 points) Find the matrix elements

a) $\left\langle lm \middle| \hat{L}^2 \middle| ll \right\rangle$,

b) $\left\langle lm \middle| \hat{L}_z \middle| ll \right\rangle$,

c) $\left\langle lm \middle| \hat{L}_x \middle| ll \right\rangle$

for arbitrary l and m. The symbol $|ll\rangle$ denotes the state in which both the orbital and magnetic quantum numbers are equal to l.

Problem 2. (30 points) Two photons, shared between Alice and Bob, are in the state $|\Psi\rangle = \frac{1}{\sqrt{2}}(|HH\rangle + i\,|VV\rangle)$. Alice measures her photon in the $\{|\alpha\rangle, \left|\frac{\pi}{2} + \alpha\right\rangle\}$ basis and detects $|\alpha\rangle$ (the state polarized at angle α to horizontal). For $\alpha \in [0, 2\pi]$, find the loci on the Bloch sphere corresponding to the states

a) detected by Alice,

b) remotely prepared at Bob's station by Alice's measurement.

You can use the template below to plot the required loci.
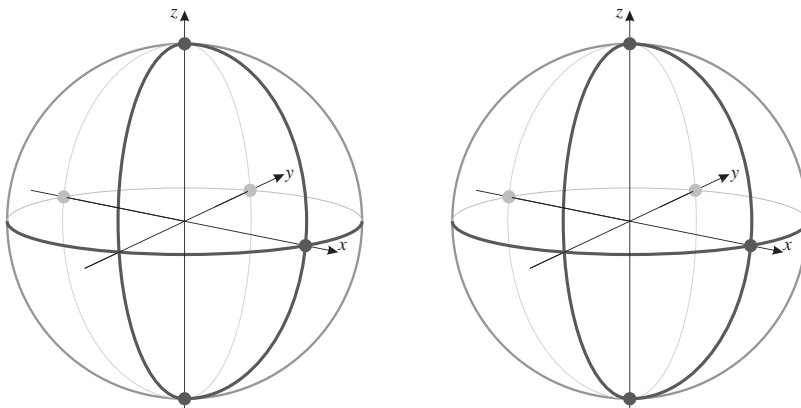
Problem 3. (30 points) Write the wavefunction of the electron in the state $|n, l = n - 1, m = n - 1\rangle$ of the hydrogen atom. You are not required to normalize your answer. For full credit, the spherical harmonic must be written explicitly in terms of elementary functions of polar angles.