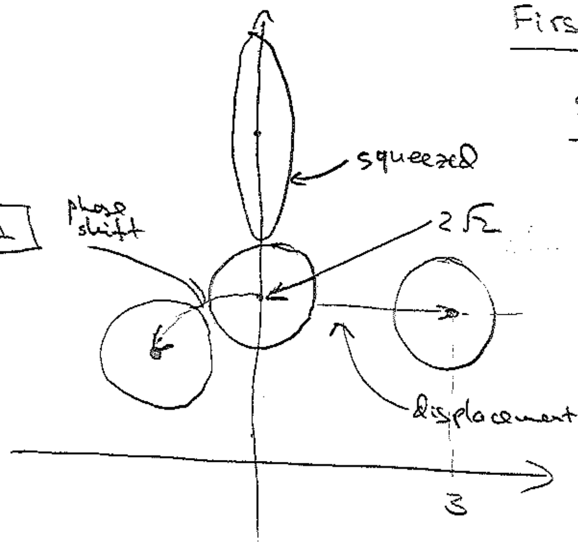
PHYS 543 Fall 2017

First midterm

Solutions

1



	$\langle x\rangle$	$\langle p\rangle$	$\langle \Delta x^2\rangle$	$\langle \Delta p^2\rangle$
original	0	$2\sqrt{2}$	$1/2$	$1/2$
displaced	3	$2\sqrt{2}$	$1/2$	$1/2$
phase shifted	-2	2	$1/2$	$1/2$
squeezed	0	$2\sqrt{2}$	$1/8$	2

2 $\left[[L_j, r_k], [L_m, p_n]\right] = (+i\hbar)^2 \left[\varepsilon_{jk\ell}\, r_\ell\, \varepsilon_{mnq}\, p_q\right]$

$= -\hbar^2\, \varepsilon_{jk\ell}\, \varepsilon_{mnq}\, (+i\hbar)\, \delta_{\ell q}$

$= -i\hbar^3\, \varepsilon_{qjk}\, \varepsilon_{qmn} = -i\hbar^3 (\delta_{jm}\delta_{kn} - \delta_{jn}\delta_{km})$

3 a) $\dot{x} = \frac{i}{\hbar}[H\ x] = i\gamma p[p\ x] + i\gamma[p\ x]\, p = 2\gamma p$

$\dot{p} = \frac{i}{\hbar}[H\ p] = 0$

$x(t) = x(0) + 2\gamma t\, p(0)$

$p(t) = p(0)$

b) $y(t) = \dfrac{x(0) + (2\gamma t - 1)\, p(0)}{\sqrt{2}} =$

$\langle y(t)\rangle = 0$

$\langle \Delta y^2(t)\rangle = \frac{1}{2}\left(\langle x(0)^2\rangle + (2\gamma t - 1)^2 \langle p(0)^2\rangle + 2(2\gamma t - 1)\langle x(0)p(0)\rangle\right)$

$= \frac{1}{2}\left(\frac{1}{2} + \frac{1}{2}(2\gamma t - 1)^2\right)$

$= \frac{1}{4}\left(4\gamma^2 t^2 - 4\gamma t + 2\right)$

Squeezing present if $4\gamma^2 t^2 - 4\gamma t < 0$, i.e. $\gamma t < 1$