PHYS 543 Fall 2017

Homework 6

Solutions

$\boxed{6.1}$ a) $\rho = |\alpha|^2 |0\rangle\langle 0| + \alpha\beta^* |0\rangle\langle 1| + \alpha^*\beta |1\rangle\langle 0| + |\beta|^2 |1\rangle\langle 1|$

$$\rho' = |\alpha|^2 |0\rangle\langle 0| + A_{10}\, \alpha\beta^* |0\rangle\langle 1| + A_{10}\, \alpha^*\beta |1\rangle\langle 0|$$
$$+ |\beta|^2 \left(A_{10}^2 |1\rangle\langle 1| + A_{11}^2 |0\rangle\langle 0|\right)$$
$$= |\alpha^2| |0\rangle\langle 0| + t\alpha\beta^* |0\rangle\langle 1| + t\alpha^*\beta |1\rangle\langle 0|$$
$$+ |\beta|^2 \left(t^2 |1\rangle\langle 1| + r^2 |0\rangle\langle 0|\right)$$
$$= \begin{pmatrix} |\alpha|^2 + (1-\eta)|\beta|^2 & \sqrt{\eta}\,\alpha\beta^* \\ \sqrt{\eta}\,\alpha^*\beta & \eta|\beta|^2 \end{pmatrix}$$

b) $1 - \eta = \gamma \quad \Rightarrow \quad \sqrt{\eta} = \sqrt{1-\gamma} \approx 1 - \gamma/2$

$$\rho' = \begin{pmatrix} |\alpha|^2 + \gamma|\beta|^2 & (1-\gamma/2)\alpha\beta^* \\ (1-\gamma/2)\alpha^*\beta & (1-\gamma)|\beta|^2 \end{pmatrix}$$

c) $pr_\rho(x) = |\alpha\, \psi_0(x) + \beta\, \psi_1(x)|^2 =$
$$= |\alpha + \sqrt{2}\beta x|^2 \frac{1}{\sqrt{\pi}} e^{-x^2}$$

$$pr_{\rho'}(x) = \langle x|\rho'|x\rangle$$
$$= \left(|\alpha|^2 + (1-\eta)|\beta|^2\right) \psi_0^2(x)$$
$$+ \sqrt{\eta}\left(\alpha^*\beta + \alpha\beta^*\right)\psi_0(x)\psi_1(x)$$
$$+ \eta|\beta|^2 \psi_1^2(x)$$
$$= \left[|\alpha|^2 + (1-\eta)|\beta|^2 + \sqrt{2\eta}\, x(\alpha^*\beta + \beta^*\alpha) + 2\eta|\beta|^2 x^2\right] \frac{1}{\pi} e^{-x^2}$$

$\boxed{6.2.}$ $|TMSV\rangle = |00\rangle + r_s|11\rangle$

$|\alpha\rangle = |0\rangle + \alpha|1\rangle$

$|TMSV\rangle \otimes |\alpha\rangle = |000\rangle + r_s|110\rangle + \alpha|001\rangle$

$$B_{23}(|TMSV\rangle \otimes |\alpha\rangle) = |000\rangle + \frac{r_s}{\sqrt{2}}|110\rangle + \frac{r_s}{\sqrt{2}}|101\rangle + \frac{\alpha}{\sqrt{2}}|001\rangle - \frac{\alpha}{\sqrt{2}}|010\rangle$$

Project onto $\langle 1|$ in mode 3:

$$\langle 1_3| B_{23}(|TMSV\rangle \otimes |\alpha\rangle) = + \frac{r_s}{\sqrt{2}}|10\rangle + \frac{\alpha}{\sqrt{2}}|00\rangle$$

Discard mode 2 $\rightarrow$ $+\frac{r_s}{\sqrt{2}}|1\rangle + \frac{\alpha}{\sqrt{2}}|0\rangle$

Normalize $\rightarrow$ $\frac{1}{\sqrt{r_s^2+\alpha^2}}\left(\alpha|0\rangle + r_s|1\rangle\right)$

$\boxed{6.3}$ $H' = \hbar\gamma(PX + XP) = i\hbar\gamma(a^{+2} - a^2)$

$$E_1^{(1)} = \langle 1 | H' | 1 \rangle = 0$$

$$E_1^{(2)} = \sum_{m \neq 1} \frac{|\langle m | H' | 1 \rangle|^2}{E_1^{(0)} - E_m^{(0)}}$$

The only nonvanishing term is $m = 3$

$$E_1^{(2)} = \frac{(\hbar\gamma)^2}{\hbar\omega} \frac{|\langle 3 | a^{+2} | 1 \rangle|^2}{2} = -3\hbar \frac{\gamma^2}{\omega}$$

$\boxed{6.4}$ $H' = \hbar\gamma\,(x_1^2 + 2x_1x_2 + x_2^2)$

$W_{\psi\psi} = \langle 1\,0 | H' | 1\,0\rangle = \hbar\gamma \langle 1\,0 | x_1^2 + x_2^2 | 1\,0\rangle = \frac{3}{2}\hbar\gamma + \frac{1}{2}\hbar\gamma = 2\hbar\gamma$

cross term vanishes

$W_{\phi\phi} = \langle 0\,1 | H' | 0\,1\rangle = W_{\psi\psi} = 2\hbar\gamma$

$W_{\psi\phi} = 2\langle 1\,0 | x_1 x_2 | 0\,1\rangle$

$= \hbar\gamma \langle 1\,0 | (a_1 + a_1^\dagger)(a_2 + a_2^\dagger) | 0\,1\rangle$

$= \hbar\gamma \langle 1\,0 | a_1^\dagger a_2 | 0\,1\rangle = \hbar\gamma$

$\Rightarrow \quad H' = \hbar\gamma \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ in the subspace $\{|1\,0\rangle, |0\,1\rangle\}$

Eigenvalues: $\hbar\gamma$, $3\hbar\gamma$

Eigenvectors: $\dfrac{|1\,0\rangle \mp |0\,1\rangle}{\sqrt{2}}$