University of Calgary
Winter semester 2017

PHYS 543: Quantum Mechanics II

Homework assignment 6

Due December 8, 2017

Problem 6.1. The superposition $|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$ of Fock states propagates through a dark glass whose energy transmissivity is equal to η.

a) Find the density matrix $\hat{\rho}'$ of the transmitted state in the Fock basis.

b) For $1 - \eta = \gamma \ll 1$, find the approximation of $\hat{\rho}'$ in the first order of γ. Observe the similarity to homework Problem 5.3.

c) From the result of part (a), find the position quadrature probability distribution $\mathrm{pr}(X)$ for the states $|\psi\rangle$ and $\hat{\rho}'$.

d) Use software to plot $\mathrm{pr}_{\hat{\rho}'}(X)$ for $\alpha = \beta = 1/\sqrt{2}$ and $\eta = 0,\ 0.5,\ 1$.

The coefficients α and β cannot be assumed real in parts (a-c).

Problem 6.2. Channel A of a two-mode squeezed vacuum state with the squeezing parameter $r_s \ll 1$ is overlapped on a symmetric ($t = r = 1/\sqrt{2}$) beam splitter with a coherent state of amplitude $\alpha \ll 1$ (Fig. 1).

a) Find the decomposition of the resulting three-mode state in the Fock basis up to the first order in r_S and α.

b) One of the outputs of the beam splitter is measured with a photon number detector while the other one is discarded. Find the state of channel B of the two-mode squeezed vacuum in the event the detector registers a single photon.

Note: For consistency, please treat the beam splitter using the same convention as in the lecture notes. Mode A in the figure is input mode 1 of the beam splitter.

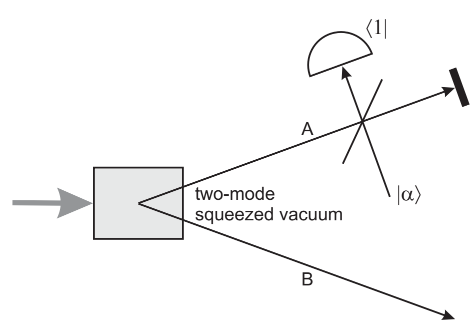

Figure 1: Illustration to Problem 6.2.

Problem 6.3. A harmonic oscillator with the zero-order Hamiltonian

$$\hat{H}_0 = \frac{\hbar\omega}{2}(X^2 + P^2)$$

is perturbed with

$$\hat{H}' = \hbar\gamma(\hat{P}\hat{X} + \hat{X}\hat{P}),$$

where $\gamma \ll \omega$. Find the corrections to the energy of the first excited level state $|1\rangle$ in the first and second orders of the perturbation theory.

Problem 6.4. The first excited level of the two-dimensional harmonic oscillator with the zero-order Hamiltonian

$$\hat{H}_0 = \frac{\hbar\omega}{2}(\hat{X}_1^2 + \hat{X}_2^2 + \hat{P}_1^2 + \hat{P}_2^2)$$

is doubly degenerate: the states $|\psi\rangle = |1, 0\rangle$ and $|\varphi\rangle = |0, 1\rangle$ have the same energy $2\hbar\omega$ (see Ex. 4.4 in the PHYS 543 lecture notes). The system is perturbed with

$$H' = \hbar\gamma(\hat{X}_1 + \hat{X}_2)^2$$

where $\gamma \ll \omega$. Find the corrections to the energies of these states and the eigenstates of the perturbed Hamiltonian in the first order of the perturbation theory.