University of Calgary
Winter semester 2017

PHYS 543: Quantum Mechanics II

Homework assignment 3

Due November 3, 2017

Problem 3.1. Consider the evolution of the spin state of a spin-1 particle under the action of a constant magnetic field $\vec{B}$ oriented along the x axis. The initial state is $|\psi(0)\rangle = |m_s = 1\rangle$.

a) Find the spin state $|\psi(t)\rangle$ as a function of time in the matrix form, in the eigenbasis of $\hat{S}_z$.

b) Find the mean values $\left\langle \hat{S}_x(t) \right\rangle, \left\langle \hat{S}_y(t) \right\rangle, \left\langle \hat{S}_z(t) \right\rangle$ and verify that they are consistent with what is expected classically.

c) The state $|\psi(t)\rangle$ is measured using a Stern-Gerlach apparatus with the magnetic field along the y axis. Find the probability for the particle to land in each of the three spots. Are the values found at one-quarter and three-quarters of the Larmor period consistent with what you would expect from part (b)?

Problem 3.2. In a Ramsey spectroscopy experiment, instead of the standard excitation pulse sequence $\left(\frac{\pi}{2}, \pi/2\right)$, the sequence

a) $\left(\frac{\pi}{2}, \theta\right)$;

b) $\left(\theta, \frac{\pi}{2}\right)$

is applied. Calculate the populations of the states $|\uparrow\rangle$ and $|\downarrow\rangle$ as a function of θ and Δt, where Δ is the detuning of the rf field and t the duration of the experiment.

Problem 3.3. Calculate all Clebsch-Gordan coefficients corresponding to the coupling between systems with $l_1 = \frac{3}{2}$ and $l_2 = \frac{1}{2}$, using the same approach as developed in the class.

- Find all possible values of l for which the Clebsch-Gordan coefficients do not vanish.
- Using the result of homework problem 2.2, write

$$|l = 2, m = 2\rangle = |m_1 = 3/2, m_2 = 1/2\rangle\,. \tag{1}$$

  Obtain the corresponding Clebsch-Gordan coefficient.

- Apply the lowering operator $\hat{L}_- = \hat{L}_{1-} + \hat{L}_{2-}$ to both sides of Eq. (1) several times to find the decompositions of all states $|l = 2, m\rangle$ into the $|m_1, m_2\rangle$ basis. Express the coefficients of these decompositions as Clebsch-Gordan coefficients.
- States $|l = 2, m = 1\rangle$ and $|l = 1, m = 1\rangle$ both decompose into states $|m_1 = 3/2, m_2 = -1/2\rangle$ and $|m_1 = 1/2, m_2 = 1/2\rangle$. Knowing the decomposition of state $|l = 2, m = 1\rangle$ from the previous calculation, find the decomposition of $|l = 1, m = 1\rangle$ that is orthogonal. The overall phase of that decomposition is a matter of convention. Choose that phase so that the coefficient in front of $|m_1 = 3/2, m_2 = -1/2\rangle$ is real and positive.
- Repeat the third step for $l = 1$.

Problem 3.4. Alice and Bob each have a spin-1 particle. The added angular momentum of the two particles is in state $|l = 2, m = -1\rangle$. Alice peforms a Stern-Gerlach measurement of her particle with the magnetic field along the x axis and detects the spin projection value $m_x = 1$.

a) What state does this measurement prepare at Bob's location? The answer should be given in the eigenbasis of Bob's $\hat{L}_z$.

b) What is the probability of this event?