University of Calgary
Winter semester 2017

PHYS 543: Quantum Mechanics II

Homework assignment 2

Due October 20, 2017

Problem 2.1. For $l = 3/2$:

a) Find the matrices of $\hat{L}_x$, $\hat{L}_y$, $\hat{L}_z$, $\hat{L}_\pm$, and $\hat{L}^2$ explicitly.

b) Verify that these matrices obey $\hat{L}_x^2 + \hat{L}_y^2 + \hat{L}_z^2 = \hat{L}^2$.

c) Determine the commutators $[\hat{L}_i, \hat{L}_j]$ in the matrix form and verify that they are consistent with the known commutation relations for the angular momentum components.

d) Find the eigenvalues and normalized eigenstates of $\hat{L}_x$.

e) As we know, the raising and lowering operators $\hat{L}_\pm$ respectively increase and decrease the eigenvalue $\hat{L}_z$ by $\hbar$. Find the matrices of the analogous operators $\hat{L}_\pm^x$ that would raise and lower the eigenstates of $\hat{L}_x$.

f) Apply $\hat{L}_+^x$ to the eigenstates of $\hat{L}_x$ and verify that their action is analogous to that of $\hat{L}_+$ on the eigenstates of $\hat{L}_z$ (up to an arbitrary phase factor that may arise from the randomness in defining the eigenstates of $\hat{L}_x$).

Problem 2.2. Consider two objects whose angular momentum states are $|l_1, m_1 = l_1\rangle$ and $|l_2, m_2 = l_2\rangle$. Show that the tensor product state $|l_1, m_1 = l_1\rangle \otimes |l_2, m_2 = l_2\rangle$ is an eigenstate of the operators $\hat{L}^2$ and $\hat{L}_z$ (where $\hat{\vec{L}} = \hat{\vec{L}}_1 + \hat{\vec{L}}_2$) with the eigenvalues corresponding to $l = m = l_1 + l_2$.
Hint: Express $\hat{L}_x$ and $\hat{L}_y$ through $L_{\pm,1}$ and $L_{\pm,2}$.

Problem 2.3. Consider the eigenstate $|lm_{\theta\phi}\rangle$ of the observable $\hat{L}_{\vec{R}_{\theta\phi}}$ (which is the projection of $\hat{\vec{L}}$ onto the vector $\vec{R}_{\theta\phi}$ with the spherical coordinates (θ, ϕ)) with the eigenvalue $m\hbar$. Find the mean values of $\hat{L}_x$, $\hat{L}_y$, $\hat{L}_z$ in this state and show that they are proportional to the projections of the vector $\vec{R}_{\theta\phi}$ onto the corresponding coordinate axes.
Hint: Change the reference frame to (x', y', z') such that the new axis z' is parallel to $\vec{R}_{\theta\phi}$ and express $\hat{L}_x$, $\hat{L}_y$ and $\hat{L}_z$ through $\hat{L}_{x'}$, $\hat{L}_{y'}$ and $\hat{L}_{z'}$.

Problem 2.4. Calculate the normalized radial wavefunction of the state $|n, l = n-2\rangle$ of the hydrogen atom with an arbitrary principal quantum number n.
Hint: $\int\limits_0^\infty x^n r^{-x} dx = n!$

Problem 2.5. Find the matrix elements (a) $\langle 100| \hat{A} |200\rangle$, (b) $\langle 100| \hat{A} |210\rangle$, (c)$\langle 100| \hat{A} |211\rangle$ of observables $\hat{A} = \hat{x}, \hat{y}, \hat{z}$ in the hydrogen atom.

Problem 2.6. Linearly polarized photons with different polarization angles α pass through a quarter-wave plate with its optic axis oriented

a) horizontally;

b) at 45°.

Find the locus of the resulting states on the Bloch sphere.