PHYS 443 Fall 2015

Midterm 2

Solutions

1 a) $a^2 + b^2 = 1$

b) $Tr_A |\Psi\rangle\langle\Psi| = \langle +|\Psi\rangle\langle\Psi|+\rangle + \langle -|\Psi\rangle\langle\Psi|-\rangle$

$$= a^2 |+\rangle\langle +| + b^2 |-\rangle\langle -|$$

$$= \frac{a^2}{2}\left(|H\rangle\langle H| + |H\rangle\langle V| + |V\rangle\langle H| + |V\rangle\langle V|\right)$$

$$+ \frac{b^2}{2}\left(|H\rangle\langle H| - |H\rangle\langle V| - |V\rangle\langle H| + |V\rangle\langle V|\right)$$

$$= \frac{a^2+b^2}{2}|H\rangle\langle H| + \frac{a^2-b^2}{2}|H\rangle\langle V| + \frac{a^2-b^2}{2}|V\rangle\langle H| + \frac{a^2+b^2}{2}|V\rangle\langle V|$$

$\rho = \begin{pmatrix} a^2 & 0 \\ 0 & b^2 \end{pmatrix}$ in diagonal basis

$\rho = \frac{1}{2}\begin{pmatrix} a^2+b^2 & a^2-b^2 \\ a^2-b^2 & a^2+b^2 \end{pmatrix}$ in canonical basis

c) $Tr\,\rho^2 = a^4 + b^4$. Fully mixed for $a = b = \frac{1}{\sqrt{2}}$, Pure for $a=1$ or $b=1$

d) $R_x = a^2 - b^2$, $R_y = 0$ $R_z = 0$ (mixture of $|+\rangle$ and $|-\rangle$)

$|R| = a^2 - b^2$, $\Theta = 90°$, $\varphi = 0$.

2 a) $\hat{H} = H_0 + \frac{A}{2}\left(\hat{J}^2 - \hat{L}^2 - \hat{S}^2\right)$

Eigenvalues $E = H_0 + \frac{A\hbar^2}{2}\left(j(j+1) - \ell(\ell+1) - s(s+1)\right)$ (where $s = \frac{1}{2}$)

For $j = \ell + \frac{1}{2}$: $E = H_0 + \frac{A}{2}\hbar^2\left((\ell+\frac{1}{2})(\ell+\frac{3}{2}) - \ell(\ell+1) - \frac{1}{2}(\frac{3}{2})\right) = H_0 + \frac{A\hbar^2}{2}\ell$

For $j = \ell - \frac{1}{2}$: $E = H_0 - \frac{A}{2}\hbar^2\left((\ell-\frac{1}{2})(\ell+\frac{1}{2}) - \ell(\ell+1) - \frac{1}{2}\cdot\frac{3}{2}\right) = H_0 - \frac{A\hbar^2}{2}(\ell+1)$

b) For $j = \ell + \frac{1}{2}$, $2j+1 = 2\ell+2$

For $j = \ell - \frac{1}{2}$, $2j+1 = 2\ell$

c) $H = H_0 + A\vec{L}\cdot\vec{S} + \gamma B(L + 2S)$

Neglect second term. Eigenvalues:

$$E_{\ell,s} = H_0 + \gamma\hbar B\,(m_\ell + 2m_s)$$

$$m_\ell = \{-\ell \ldots \ell\}$$

$$2m_s = \{-1, +1\}$$

For $2m_s = +1$, slopes $\{\gamma\hbar(-\ell+1) \ldots \gamma\hbar(\ell+1)\}$

For $2m_s = -1$, slopes $\{\gamma\hbar(-\ell-1) \ldots \gamma\hbar(\ell-1)\}$

All possible slopes: $\{\gamma\hbar(-\ell-1) \ldots \gamma\hbar(\ell+1)\}$

For slopes $\{\gamma\hbar(-\ell-1), \gamma\hbar(-\ell), \gamma\hbar\ell, \gamma\hbar(\ell+1)\}$:
one corresponding eigenstate

For other slopes, two corresponding eigenstates.