

Midterm 1

Solutions

1) a) $[L_+, L_-] = [L_z, -iL_x] + [iL_x, L_z] = -i(i\hbar L_y) + i(-i\hbar L_y) = 2\hbar L_y$

b) $[L_+, y] = [L_z, y] + i[L_x, y] = -i\hbar x + i(i\hbar z) = -i\hbar x - \hbar z$

2) $h = 2, \ell = 1, m = 0$

a) $\psi(r, \theta, \varphi) = R_{21}(r, \theta) Y_{10}(\theta, \varphi) =$

$$= \frac{1}{\sqrt{24}} a^{-5/2} r e^{-r/2a} \sqrt{\frac{3}{4\pi}} \cos \theta = \frac{1}{\sqrt{32\pi}} a^{-5/2} r e^{-r/2a} \cos \theta$$

b) $|200 \frac{1}{2}\rangle, |200 -\frac{1}{2}\rangle, |211 \frac{1}{2}\rangle, |211 -\frac{1}{2}\rangle$

$|210 \frac{1}{2}\rangle, |210 -\frac{1}{2}\rangle, |21-1 \frac{1}{2}\rangle, |21-1 -\frac{1}{2}\rangle$

3) a) $\hat{H} = -\vec{\mu} \cdot \vec{B} = -\gamma \vec{S} \cdot \vec{B} = -\frac{g\mu_B}{2m} (S_x B_x + S_z B_z)$

$$= -\frac{g\mu_B \hbar}{2m} \left(\frac{B}{\sqrt{2}} \sigma_x + \frac{B}{\sqrt{2}} \sigma_z \right) = -\frac{g\mu_B \hbar B}{4\sqrt{2}m} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad (L)$$

b) one-half of Larmor period

$$\frac{T}{2} = \frac{\pi}{\gamma B} = \frac{\pi}{g\mu_B B}$$

c) $\begin{cases} B_x = \frac{B}{\sqrt{2}} = \frac{\sqrt{2}}{2} B \\ \sqrt{2} = \frac{\sqrt{2}}{2} B_{rt} \end{cases} \Rightarrow B_{rt} = B\sqrt{2}$