University of Calgary
Fall semester 2015

PHYS 543: Quantum Mechanics II

Homework assignment 1

Due Monday, September 21, 2015

Problem 6.1. Find the general form of the commutator $[\hat{L}_j, [\hat{L}_k, \hat{r}_l]]$. Check your answer by specific examples: $[\hat{L}_x, [\hat{L}_y, \hat{r}_z]]$, $[\hat{L}_x, [\hat{L}_x, \hat{r}_z]]$ and $[\hat{L}_x, [\hat{L}_z, \hat{r}_x]]$.

Problem 6.2. As discussed in class, the raising and lowering operators $\hat{L}_\pm$ respectively increase and decrease the eigenvalue $\hat{L}_z$ by $\hbar$.

a) Construct similar operators $\hat{L}^x_\pm$ for the eigenstates of $\hat{L}_x$

b) Find their matrices for the subspace with $l = 1$ in the eigenbasis of $\hat{L}_z$.

c) Find the eigenstates of $\hat{L}_x$ in the matrix form in the eigenbasis of $\hat{L}_z$.

d) Apply $\hat{L}^x_\pm$ to the eigenstates of $\hat{L}_x$ found in part (c) and verify that their action is analogous to that of $\hat{L}_\pm$ on the eigenstates of $\hat{L}_z$.

Problem 6.3. A general expression for the spherical harmonics is

$$Y_l^m(\theta,\phi) = \mathcal{N}_l \sqrt{\frac{(l+m)!}{(l-m)!}} \sin^{-m}\theta \frac{d^{l-m}}{d(\cos\theta)^{l-m}} \sin^{2l}\theta e^{im\phi}, \tag{1}$$

where

$$\mathcal{N}_l = (-1)^l \sqrt{\frac{2l+1}{4\pi}} \frac{1}{2^l l!} \tag{2}$$

is the normalization factor.

a) Calculate $Y_1^1(\theta,\phi)$ according to Eq. (1).

b) Verify that the application of the raising operator to this wavefunction gives zero.

c) Verify the correctness of the normalization factor for $Y_1^1(\theta,\phi)$.

d) Verify that $Y_1^1(\theta,\phi)$ represents an eigenstate of $\hat{L}^2$ with eigenvalue $2\hbar^2$ by applying operator $\hat{L}^2$ in the position basis.

e) Apply the lowering operator to state $Y_1^1(\theta,\phi)$ to find $Y_1^0(\theta,\phi)$ and verify consistency with Eq. (1).

Hint: a general solution for arbitrary l is given in the lecture notes, but here you must show the full calculation for a specific case $l = 1$.

Problem 6.4. A pair of electrons, shared between Alice and Bob, are prepared in an entangled spin state

$$\left|\Psi^-\right\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right),$$

where $|\uparrow\rangle = |m_s = 1/2\rangle$ and $|\downarrow\rangle = |m_s = -1/2\rangle$. Alice measures the projection of of her electron's spin onto vector $\vec{R}_{\theta,\phi}$ defined by spherical angles (θ,ϕ). Find the probability of each possible outcome of this measurement and the resulting state of Bob's electron.