Thin Lenses and Optical Instruments

Equipment

1 meter optical bench with 5 moveable mounts, lighted test object, 3 lens holders, screen, 2 short focal length double convex lenses (L1, E1), 2 medium focal length double convex lenses (L2 and L3), 1 long focal length double convex lens (L4), 1 double concave lens (L5), diopter gauge, meter stick, ruler, flashlight.

Purpose

To observe the operation of thin lenses and gain experience with the placement and alignment of optical components. To examine and measure real and virtual images in simple optical systems. To measure the focal lengths of double convex and double concave lenses. To understand the operation of simple optical instruments and to construct a simple microscope and telescope.

Theory

An understanding of lenses as **converging** or **diverging** and a classification of their surfaces as **concave**, **convex**, or **planar** is needed for this experiment. The relationship between object distance s_o, image distance s_i, and lens focal length, *f*, is used in several parts of the experiment. The formula relating these quantities is the **thin lens equation**:

$$\frac{1}{s_i}+\frac{1}{s_o}=\frac{1}{f}. \qquad (1)$$

A second formula that is used in parts of this experiment is the **magnification** of an image. For an object height, y_o, and an image height, y_i, the magnification, *M*, is given by

$$|M|=\left|\frac{y_i}{y_o}\right|=\left|\frac{s_i}{s_o}\right|. \qquad (2)$$

A significant portion of the experiment involves measuring the **focal lengths** of different lenses. An instrument commonly used for this purpose is the **diopter gauge**. The principle of operation of the diopter gauge is based on the **lensmaker equation** (2). Suppose a thin lens immersed in air is made from a material with **index of refraction**, *n*. One side of the lens has **radius of curvature**, R_1, and the other side has a radius of curvature, R_2. The focal length of the lens is given by the lens maker's equation

$$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right). \qquad (3)$$

For lenses that are double convex, R_1 is positive and R_2 is negative, so equation (3) implies that this type of lens has positive focal length. For lenses that are double concave, R_1 is negative and R_2 is positive, so equation (3) implies that this type of lens has negative focal length. If one of the surfaces is flat the radius of curvature will be infinite. The reciprocal is then zero and the lens maker's formula implies that this surface does not contribute anything towards determining the focal length. A lens with both sides flat (a window) has an infinite focal length.

In many optics formulae (including the lens maker's equation), the focal length only appears as a reciprocal. It therefore makes sense to define a unit of reciprocal focal length. The **power**, *D*, of a lens is defined as the reciprocal of the focal length of the lens in meters. The unit of lens power is called the **diopter** (*D*). Since converging lenses have positive focal lengths and diverging lenses have negative focal lengths, the power of a lens is positive for converging lenses and negative for diverging lenses. For example, a converging lens with a focal length of one meter has a power of one diopter (1 diopter = $1 m^{-1}$). A diverging lens with a focal length of 0.2 meters has a power of -5.0 diopters. A flat piece of window glass does not converge or diverge incoming light rays, it has a power of zero diopters. The lens maker's formula can be rewritten in terms of dioptric power to get

$$D = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right). \qquad (4)$$

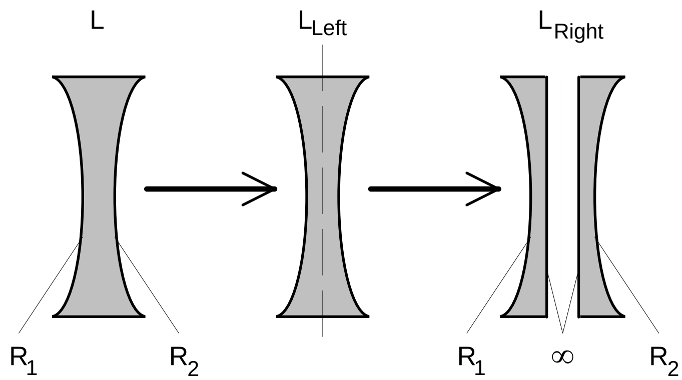

Figure 1

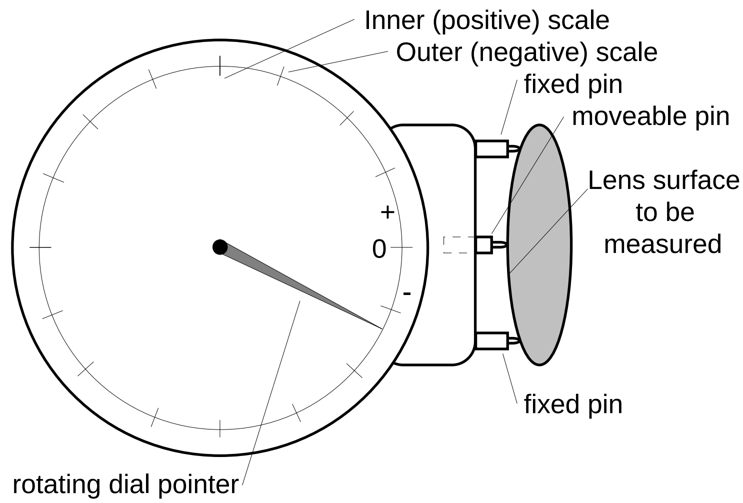

Figure 2

As shown in Figure 1, imagine a lens, L, as being split in half to form a combination of two lenses, L_{Left} and L_{Right}, each of which is flat on one side. The two lenses, L_{Left} and L_{Right}, are in contact on their flat sides and touching. Lens L has a radius of curvature R_1 on the left and R_2 on the right. The power of lens L is given by equation (4). Lens L_{Left} has a radius of curvature R_1 on the left and ∞ on the right. Similarly, Lens L_{Right} has radius of curvature ∞ on the left and R_2 on the right. Let the power of lens L_{Left} be D_{Left} and the power of lens L_{Right} be D_{Right}. Then by the lens maker's formula the power of each half lens is given by

$$D_{Left} = \frac{(n-1)}{R_1} \qquad (5)$$

and

$$D_{Right} = -\frac{(n-1)}{R_2}. \qquad (6)$$

Observe that the sum of equations (5) and (6) yields equation (4). This means that the **power of a lens is equal to the sum of the powers of its two surfaces**. If an instrument could measure the power of a single surface then the focal length of a lens could be measured by adding together the power of each lens surface. This is one of the principles upon which the operation of a diopter gauge is based.

Figure 2 shows a diagram of a diopter gauge measuring one surface of a lens. A diopter gauge measures the power of a single surface by determining the radius of curvature. The gauge is held against the surface of the lens so that it is perpendicular to the surface and the two fixed pins are touching the glass. If it is assumed that the surface is a section from a circle then the amount that the moveable pin is compressed determines the radius of curvature of the surface. If the index of refraction of the material is known, the dioptric power of the surface can be found from equation (5). The diopter gauges used in this experiment are calibrated for **crown glass** with an index of refraction of 1.525. The scales read off the power of the surface directly in diopters when the lens material is crown glass.

The procedure for measuring the power of a lens with the diopter gauge is as follows. Determine whether the surface is convex or concave by holding a straight edge against the lens. Do not scratch the surface or leave fingerprints on it. If the surface is convex read from the black inner scale. If the surface is concave read from the red outer scale. A flat surface will read zero on both scales. Hold the gauge perpendicular to the surface so that all three pins touch the surface. The pins are rounded and treated so that they do not scratch the glass. Read the power of the surface in diopters. Repeat the procedure for the other surface. The total power of the lens is sum of the two diopter readings. The focal length is then the inverse of this number in meters.

The rules for adding powers of surfaces together to get the total power of a lens are simple. Convex surfaces have positive power, concave surfaces have negative power, and flat surfaces have zero power. The total lens power is the sum of the power of the two surfaces. Examples for various types of lenses are shown in Figure 3.

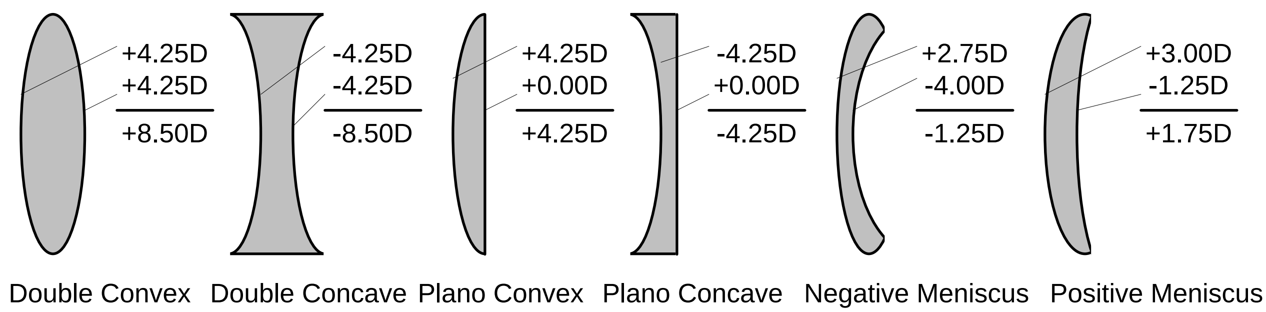

Figure 3

Another method useful for determining the focal length of a lens is called the **displacement method**. Suppose an object and a screen are located a fixed distance, b, apart. The converging lens whose focal length is to be measured is in between. As the lens is moved to different locations between the object and the screen it is found that a real image is focused on the screen only when the lens is in two specific positions. At one position the image is larger than the object and at the other position the image is smaller than the object.

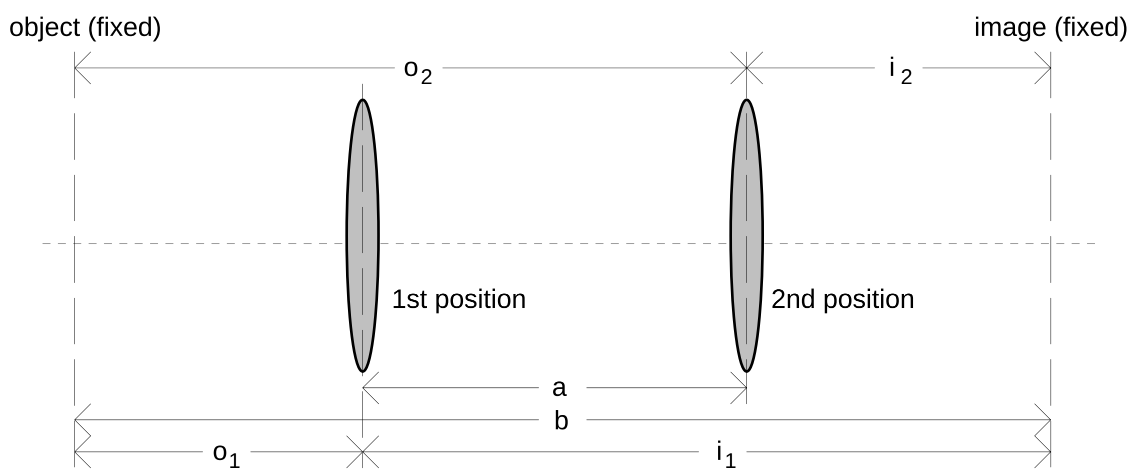

Figure 4

As in Figure 4, define *a* as the separation between the two positions of the lens. Once *a* and *b* are known, the focal length can be found by the formula

$$f = \frac{b^2 - a^2}{4b}. \qquad (7)$$

The smallest possible value *a* can have is zero. Substituting *a*≥*0* into equation (7) shows that the longest focal length that can be measured for a given *b* is *b/4*. For the optical bench used in this experiment, this limits the measurable focal length to anything below 30 cm.

An advantage of this method is that it gives accurate results because the distance, *a*, is independent of the thickness of the lens and the position of the lens in the lens holder. Similarly, the distance, *b*, between object and screen can be measured accurately so that offsets of the screen and object can be eliminated. Furthermore, *b* needs to be measured only once since object and screen remain fixed.

A second advantage of this method is that it permits the measurement of sizes and positions of inaccessible objects such as the filament inside a bulb or a virtual image located behind a lens. This will be used in this experiment to measure the position and magnification of a virtual image produced by a lens used as a magnifier. It will also be used in this experiment to measure the position of a virtual image generated by a diverging lens.

Imagine replacing the object in Figure 4 with some sort of lens system that produces a virtual image, such as a concave lens. Apply the displacement method with a lens of known focal length to get a value for *a*. Solving equation (7) for *b* then gives a value for the position of the virtual image.

The size of the virtual image can also be measured. Suppose the virtual image has size, *y*. Let y_1 and y_2 be the sizes of the real image on the screen for the two possible positions of the lens. Let M_1 and M_2 be the magnifications at the two lens positions. M_1 and M_2 are reciprocals. So

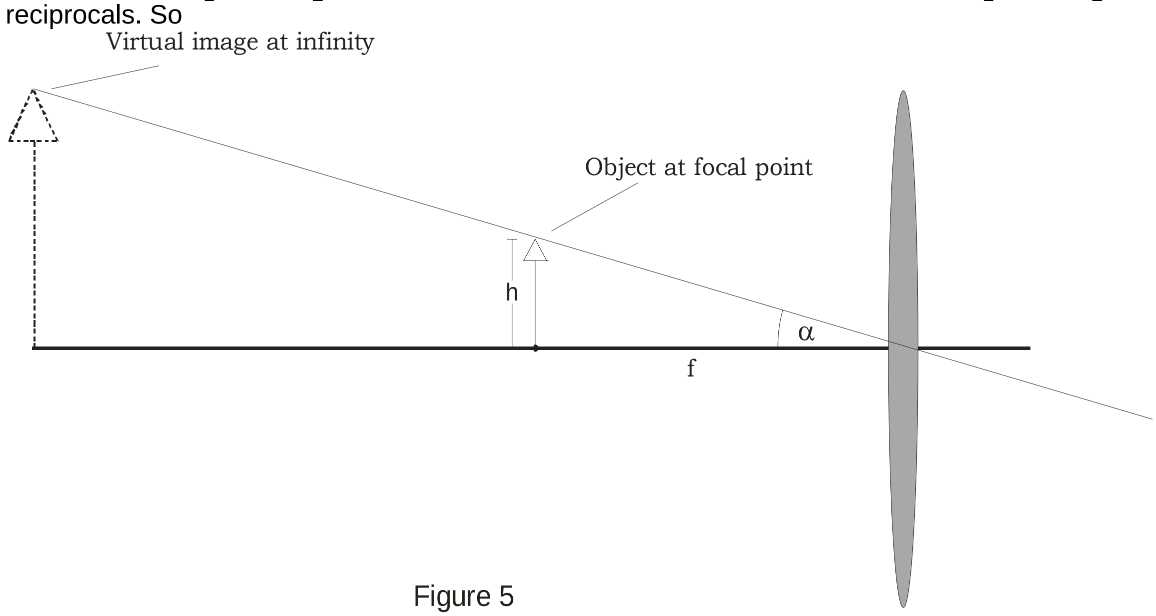

Figure 5

$$1 = M_1 M_2 = \left(-\frac{y_1}{y}\right)\left(-\frac{y_2}{y}\right) = \frac{y_1 y_2}{y^2}. \qquad (8)$$

Therefore, the virtual image size, *y*, is

$$y = \sqrt{y_1 y_2}\ . \qquad (9)$$

Equation (9) gives the size of the image being tested from the measured sizes of the images on the screen.

An **optical instrument** is a combination of optical elements that creates a magnified image of a small or distant object. One of the simplest optical instruments is the **simple**

magnifier or **magnifying glass**, which is a single converging lens which produces a virtual, magnified image of an object placed on or just inside the focal fange of the lens (figure 5).

To understand the function of a magnifying glass, we first analyze a naked eye observer who is viewing a small object. Such an observer will try to make an object appear larger by bringing it closer to his or her eye. At a certain point, however, the object will become blurry as the eye can no longer accommodate the strong focus required to cast an image on the retina. This point is called the **near point** and is defined to be 25 cm (although of course the exact position at which this occurs will vary from observer to observer). The object at the near point observed with a naked eye subtends an angle $\alpha_0 = \tan^{-1} h_0 / 25\,\mathrm{cm}$ (figure 5).

If we observe an object through a magnifying glass, we are in fact looking at its a virtual, magnified image created near infinity (recall that for optical purposes, infinity means at a distance much larger than the scale of the experiment). The angular size of the image is the same as that of the object: $\tan\alpha = h_o / f$ (figure 5). However, because the image is far away, the eye will have no problem focusing on it. In this way, if f < 25 cm, the magnifying glass allows us to increase the angular size of the image by letting us bring the object closer to the eye.

The **angular magnification** of the magnifying glass is given by the ratio of the apparent image's angle α to that of the angle α_0 made by the object when viewed by the unaided eye:

$$M_a = \frac{\tan\alpha}{\tan\alpha_0} = \frac{h_o / f}{h_o / 25cm} = \frac{25\,cm}{f}\,. \tag{10}$$

Despite the advantage of being simple, the magnifying power of the simple magnifier is limited by aberrations. In order to achieve high magnification of a nearby object with fewer aberrations, the **compound microscope** is used, achieving magnifications much greater than that of the simple magnifier. A simple compound microscope is shown in figure 6. The **objective lens** produces a real, inverted, and magnified image known as the intermediate image. A simple magnifier is then used to produce a magnified virtual image of the intermediate image.

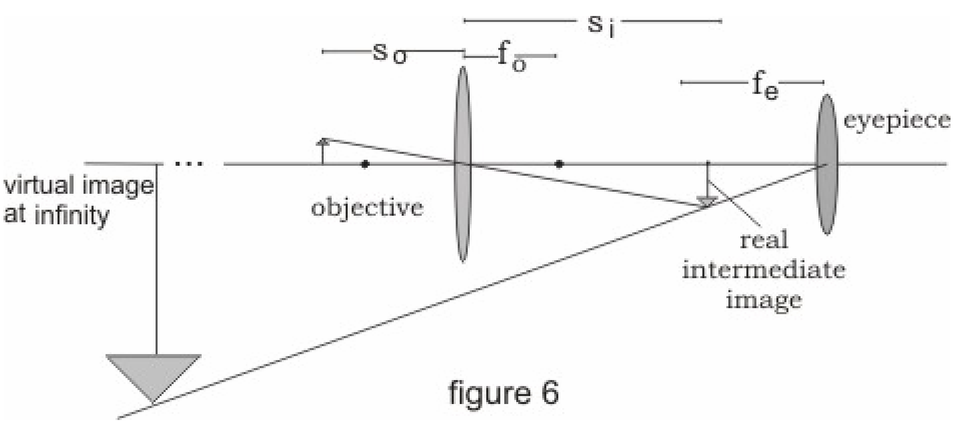

figure 6

Say we wish to magnify an object of height h_o with a compound microscope as in figure 6. How much magnification do we see? The height of the intermediate image is $h_i = h_o \frac{s_i}{s_o}$, and so the magnification due to the objective lens is $M_{obj} = s_i / s_o$. The image is then put through a simple magnifier with magnification given by equation (10). The total magnification is then given by the product of the two magnifications:

$$M = M_{obj} \times M_a = \frac{s_i}{s_o} \times \frac{25cm}{f_e} \tag{11}$$

In contrast to the microscope which magnifies the image of a near-by object with a generally high powered objective, the purpose of the telescope is to increase the retinal image of a distant object. Figure 7 displays the principles of a **Keplerian telescope**. The object is assumed to be at infinity (which in terms of optics means at least a few meters) so that incoming rays are parallel to one another. The objective lens forms a real, inverted intermediate image. From equation (1), we see that if $s_o \to \infty$, the $s_i \approx f$, so the image is formed at the focal point of the objective. The eyepiece once again produces, at infinity, a virtual image of the intermediate image. Note that for distant objects, the objective lens and magnifying eyepiece are nearly **confocal.**

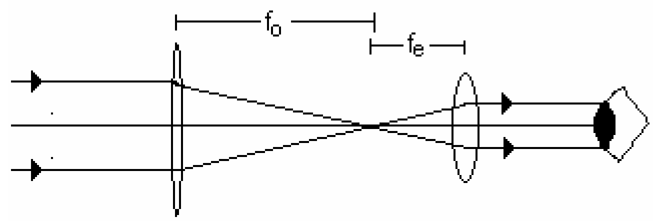

Figure 7

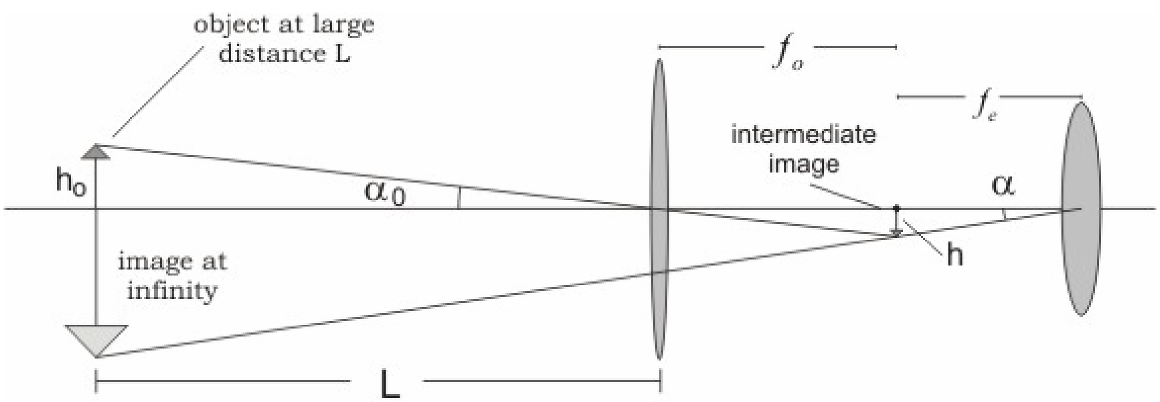

figure 8

To arrive at an expression for the magnification of a Keplerian telescope observe figure 8 where a large object of height h_o is viewed at a great distance L. Since $h_o << L$, we have that $\alpha \approx \sin\alpha = h_o / L$. Because $s_o = L$ is large, the intermediate image is located at $s_{ii} \approx f_o$ [according to equation (1)] and has the size $h_{ii} = hf_e / L$ [according to equation (2)].

The intermediate image is placed near the focal point of with the eyepiece, and we look at it as if through the magnifying glass, seeing a magnified virtual image of the intermediate image. Following the magnifying glass argument, we see that the angular size of this image (from the viewpoint of the observer) is $\alpha = h_{ii} / f_e = hf_o /(Lf_e)$, we can then write the magnification of the Celeriac telescope:

$$M \approx \frac{\alpha}{\alpha_0} = \frac{f_o}{f_e} \qquad (12)$$

Figure 9 shows the experimental layout. Optical components such as lenses, screens and test objects can be placed into holders that slide along on an optical bench. The optical bench ensures that all the components are aligned correctly. The side of the optical bench has a scale that is used to measure or set the distances between various components. A lighted test object is used to make image location clearly visible.

Five converging lenses ranging in focal lengths from short (L1) and (E1), through medium (L2 and L3), to long (L4), and one diverging lens (L5) are supplied for use in the experiment.

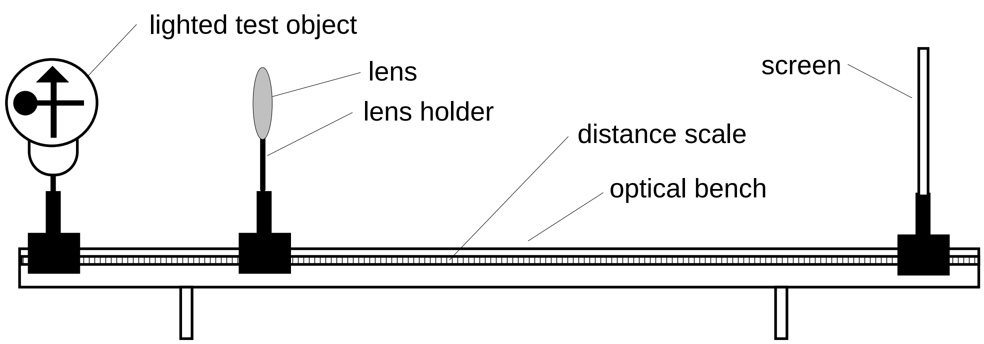

Figure 9

Experimental Procedure

1) The laboratory lights should be dimmed for the duration of this experiment. It is possible to determine the focal length of converging lenses in several ways. This experiment begins by examining some different methods for measuring focal lengths of converging lenses. First, determine the focal lengths of lenses L1, L2, L3, and L4 by the method of placing an object at infinity. For a converging lens, when an object is at infinity the image lies at the focal point with zero size. As an approximation to an object at infinity, use a lighted object on the other side of the laboratory. Point the optical bench at the lighted object. The lens to be measured is placed into a holder on the optical bench. Using the screen, locate the position of the real image and thereby the focal length. After these measurements it may be useful to place the lenses on a labeled sheet of paper with the relevant symbols L1 through L5 so that the lenses do not get mixed up. L1 is the shortest focal length converging lens and L4 the longest, with L2 and L3 intermediate. L5 is the diverging lens.

2) Determine the focal lengths of lenses L1, L2, L3, L4, and L5 with the diopter gauge.

3) Use the displacement method to measure the focal length of lens L2. Place the screen at a scale reading of 0 on the optical bench and the lighted object at a scale reading of about 100-120 on the optical bench. With lens L2 between the object and the screen locate the two positions where a real image is formed on the screen. Note that the value of b is not just the difference between the readings on the optical bench because the screen and the object are offset from the bench sliders by different distances.

4) The object distance, o, and image distance, i, are related by equation (1). For at least five different object distances, measure the image distance for the real image produced by lens L3 on the screen. The focal length of L3 can be found by plotting a graph of $1/i$ versus $1/o$.

5) Having measured the focal lengths of some converging lenses by several different methods we now turn our attention to measuring the focal length of a diverging lens using two different methods. Place the screen at a scale reading of zero on the optical bench and the lighted object at a scale reading of 115 on the optical bench. Put lens L5 in a lens holder and position it 10 to 20cm from the lighted object. A glance into L5 shows a virtual image at a position somewhere behind lens L5 that is smaller than the object.

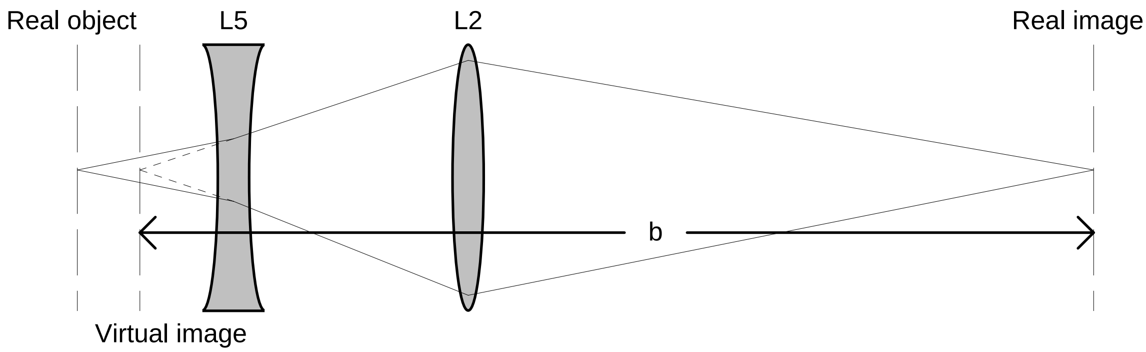

Figure 10

Select L2 as a lens with known focal length, *f*, and place it on the optical bench between L5 and the screen. Use the displacement method to find a value for *b*. This gives a value for the distance of the virtual image, *i*, from lens L5. Equation (1) can then be used to find the focal length of L5 since the object distance, *o*, between the lighted test object and L5 is readily measured.

6) The focal length of a diverging lens can also be found by measuring how far a real image shifts when the diverging lens is added to the system. As shown in Figure 11, suppose a converging lens like L1 is used to form a real image on a screen at point R. When diverging lens L5 is placed between L1 and the screen, the real image is no longer focused on the screen. The focal point has moved further away so that a real image appears on a screen at point S instead. Lens L5 diverges the light rays so they meet at a point further away.

This shift in focal length can be used to obtain a value for the focal length of L5. When L5 is inserted the real image at R disappears. Instead, lens L5 generates a real image at the new position S from the **virtual object** at point R. The image distance is the distance from L5 to S (it is positive). The object distance is the distance from L5 to R (it is negative). Equation (1) can be used to determine the focal length of L5.

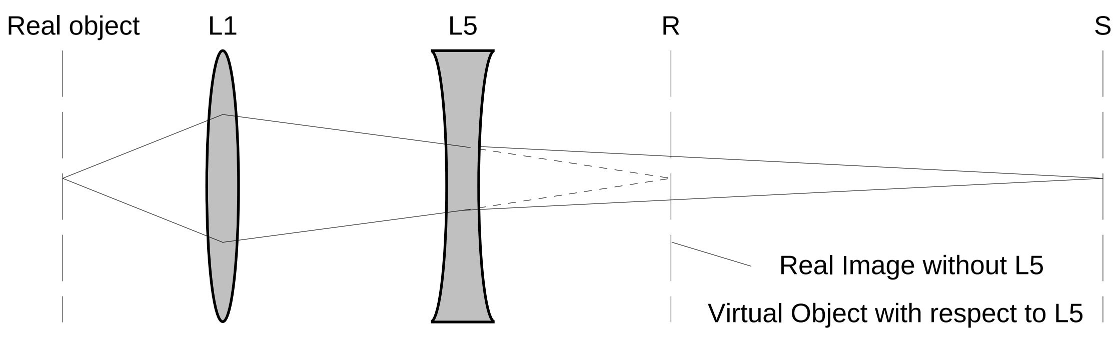

Figure 11

7) Attach the microscope grid to the dark side of the screen and observe it through lens L2 as the magnifying glass. First bring the lens close to the screen, you will see a slightly magnified virtual image. Then gradually move the lens away from the screen. Keep moving until your eyes can no longer focus on the image; this means you have reached the focal distance (in fact, you are probably still a couple of centimeters within the focal range; when you are too close to the focal points, aberrations will come into play and distort your imaging). Verify this by a direct measurement. Determine the magnification as shown in figure 12 (count the number of lines you see with the naked eye that fit in between three or four lines of the magnified image) and compare it to that predicted by equation (10). f_o

8) Construct a compound microscope. First, create a real, magnified intermediate image of the illuminated source on the screen using lens L1 as shown in figure 6. Determine the magnification by direct measurement. Second, set up the magnifying glass (step 7) to observe the microscope grid on the back side of the screen. Determine the magnification. Third, remove the screen and observe a magnified image of the source through the magnifying glass. Fourth, replace the illuminated source by the screen with the microscope grid. Determine the magnification using the scheme in figure 12 and compare it with equation (11).

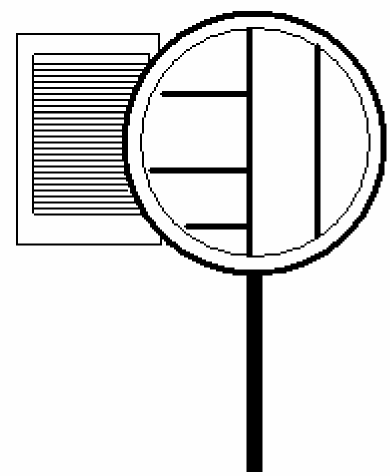

Figure 12

9) Using lenses L1 and L3, construct a Keplerian telescope with figure 7 as your guide. Question: for a positive greater than 1 magnification, which lens will you use as the objective? Make sure the focal length of L1 and L3 overlap as in figure 7. Tape the telescope grid sheet to the opposite wall and peer through your telescope with one eye. Determine the magnification from equation (12). How does your measured value compare with equation (12)? What is the significance of the negative sign in the magnification? You may have to experiment to find the right lens separations for your telescope. You should see a clear image with a visible magnification when the lenses are properly aligned.

Error Analysis

One source of error in this experiment comes from uncertainties in positioning the lenses. The optical bench permits reasonably accurate positioning of lenses along the bench. However, the height at which the lenses are placed is not precisely adjustable. Also, the orientation of the lenses should be perpendicular to the optical bench and parallel to the lighted object. Adjustments in these directions are not available on the apparatus except by rough positioning of the lens inside the lens holder.

A second source of error in this experiment is due to aberrations in the lenses themselves. A full discussion of the deviations of physical glass lenses from the theoretical ideal is beyond the level of this course. However, it can easily be seen that the quality of images at the edges is quite different from image quality at the center. This suggests that the lenses used in this experiment have different properties at their edges than at their centers. One reason for this difference is that the lenses have different thicknesses at the edge and at the center. Many of the optics formulas used in this experiment assume that the lenses are thin to partly remove this difficulty.

Thirdly, there is the issue of determining when an image is properly in focus. When a real image is observed on a screen, there is a certain tolerance in the position of all instruments within which the image appears focused. This tolerance should be determined experimentally and included in your analysis.

Finally, there is a huge systematic error in determining the magnification using the grid method in steps 7-9. You should independently determine how this error translates into your value for the magnification and what the possible ways of reducing it are.

To be included in your lab report

1. The values for the focal length of each lens measured in step 1.

2. The diopter power and corresponding focal length of the lenses from step 2.

3. The values of *a* and *b* from the displacement method in step 3, along with the corresponding focal length.

4. The plot of step 4, with uncertainties, along with the slope and corresponding focal length.

5. The focal lengths from steps 5 and 6. Include the calculation performed in step 6.

6. Derive equation (7). Show that $M_1 M_2 = 1$ in equation (8).

7. The magnification measured in step 7, its theoretical prediction, error analysis.

8. A diagram of your microscope with all positions displayed. Measured and calculated magnifications from substeps 1, 2, and 4 of step 8. Error analysis.

9. A sketch of your telescope. Show a calculation of the theoretical value for magnification, and state your measured value with an explanation of the uncertainties involved.

10. What kind of aberrations have you observed in this experiment?

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