Reflection and Refraction

Equipment

Acrylic block set, plane-concave-convex universal mirror, cork board, cork board stand, pins, flashlight, protractor, ruler, mirror worksheet, rectangular block worksheet, equilateral prism worksheet, converging lens worksheet.

Preparation

Review the laws of reflection and refraction and Snell's law. Understand the concept of total internal reflection. Review the geometry of a prism. Know the lensmaker's equation.

Goals of the Experiment

To study and observe the laws of reflection and refraction. To understand and practice optical ray tracing. To observe the operation of mirrors and prisms. To examine light travel in rectangles and equilateral triangles using the laws of reflection and refraction. To verify the lensmaker's equation.

Theory

Materials transparent to light are called **optical materials**. It has been found that the speed of light varies in different optical materials. The speed of light is highest in a vacuum. In air the speed is slightly slower. In a solid optical material like glass the speed of light can be less than half of the speed of light in free space. This speed of light difference is an important property of optical materials. The **index of refraction,** or **refractive index**, of an optical material is defined as the ratio of the speed of light in a vacuum to the speed of light in the material. The refractive index of empty space is 1.0, while the refractive index of all other optical materials is greater than 1.0. The refractive index of air is about 1.0003 while the index of refraction of the acrylic plastic used in this experiment is 1.49 .

When a ray of light passes between two optical materials, part of the ray is **reflected** and stays in the first medium while the remaining ray is **refracted** and passes into the second medium. Figure 1 shows a ray of light crossing an optical boundary. Let the index of refraction of the first medium be n and the second medium be n'. A line perpendicular to the boundary called the **surface normal**, N, is used to measure the angles of the light rays. The angle the incident ray makes with the normal is called the **angle of incidence**, φ. The angle the reflected ray makes with the normal is called the **angle of reflection** and the angle the refracted ray makes is called the **angle of refraction**, φ'. It has been found that the behaviour of reflected and refracted rays can each be described by a law.
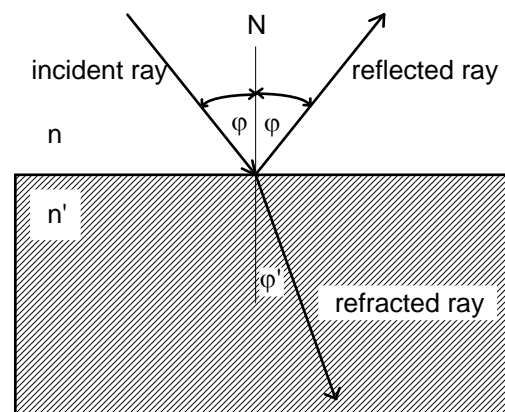

Figure 1

The **law of reflection** states that *"the angle of incidence equals the angle of reflection and the reflected ray is on the opposite side of the normal from the incident ray and the incident ray, surface normal, and reflected ray all lie in the same plane"*. The **law of refraction** states that *"the sine of the angle of incidence and the the sine of the angle of refraction are in constant ratio to each other and the refracted ray lies on the opposite side of the normal from the incident ray and the incident ray, surface normal, and refracted ray all lie in the same plane"*. The law of

refraction is also known as **Snell's law**. The constant in Snell's law is the ratio of the refractive indices of the two materials n and n'. The law of refraction is usually written as

$$\frac{\sin \varphi}{\sin \varphi'} = \frac{n'}{n}. \tag{1}$$

There are three possible cases; $n < n'$, $n = n'$, and $n > n'$.

When $n < n'$, the light rays pass from a rarified medium like air to a denser medium like glass. The reflected portion of the light remains in the rarified medium. This is called **external reflection**. As expected from the law of reflection, the incident ray and the reflected ray both are at the same angle with respect to the normal. The refracted part of the light enters the glass with an angle φ' that is determined by the law of refraction. For the smallest possible incident angle of $0°$ the refracted ray is also at $0°$. For other incident angles, as long as $n < n'$, the angle of refraction will always be **smaller** than the incident angle.

The case $n = n'$ happens when light passes through a boundary between two materials with the same index of refraction or through a single material without a boundary. Here, the angle of incidence always equals the angle of refraction. So the path of the incident ray and the path of the refracted ray are identical. As a ray of light travels through a single medium it can be imagined that there is an imaginary boundary with $n = n'$ at every point along the path of the light ray. At every boundary, the refracted ray is in the same path as the incident ray. It appears as if the ray travels through the material in a straight line. This explains why light rays travel in straight lines and cast shadows.

The case of $n > n'$ happens when light from a denser medium like acrylic enters a rarer medium like air. The reflected part of the light stays inside the acrylic. This is called **internal reflection** and obeys the law of reflection as expected. The refracted part of the incident ray leaves the acrylic and enters the air with an angle φ' that is found from the law of refraction. As with the other two cases, an incident angle of $0°$ refracts to an angle of $0°$. For all other incident angles the refracted angle will be **larger** than the incident angle. At the critical angle, φ'_c, given by

$$\varphi'_c = \sin^{-1}\left(\frac{n}{n'}\right). \tag{2}$$

the refracted ray has an angle of $90°$. For acrylic with an index of refraction of 1.49, the critical angle is $42.16°$. At incident angles above the critical angle there is no refraction. Instead, the entire ray reflects inside the acrylic. This is called **total internal reflection**.

The laws of reflection and refraction can be used to examine the passage of light through a plate with plane parallel surfaces. Figure 2 below shows an incident ray and the reflected and refracted rays at the boundaries between the air and the acrylic. The light ray emerges from the plate parallel to the original ray but laterally displaced from it.

Incident ray AB partly (about 5%) reflects externally to generate the reflected ray BG. The rest of the incident light refracts to ray BC. At the other end of the plate, ray BC internally reflects to give ray CE and refracts to produce ray CD, the exiting light. Ray CE behaves similarly and yields a refracted ray and an internally reflected ray which in turn gives a refracted and reflected component and so on. Note that because the sides of the plate are parallel,
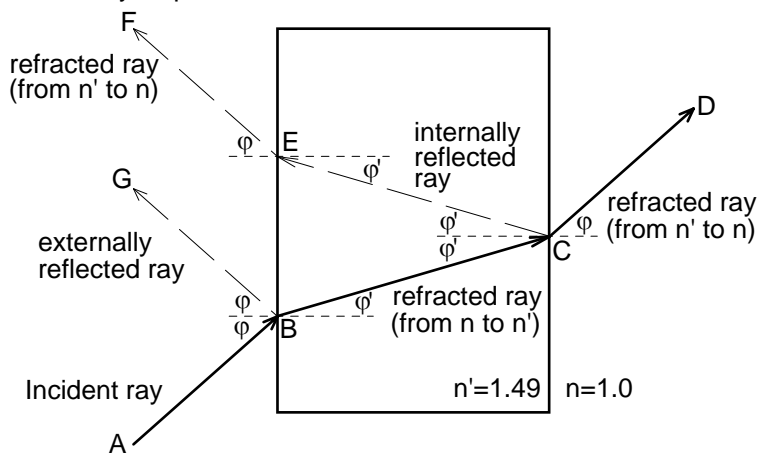

Figure 2

rays AB and CD are parallel, rays BG and EF are parallel, all internal angles are equal to the refraction angle φ', and all external angles are equal to the incident angle φ.

If the sides of the plate are not parallel, the deviation produced by the first refraction is not undone by the second refraction. So the incident rays and exiting rays are not parallel. A **prism** takes advantage of this fact by inclining two surfaces at an angle α to achieve a desired amount of deviation, δ. Figure 3 shows an **equilateral prism.** Here the sides are all inclined at α = 60° to each other.

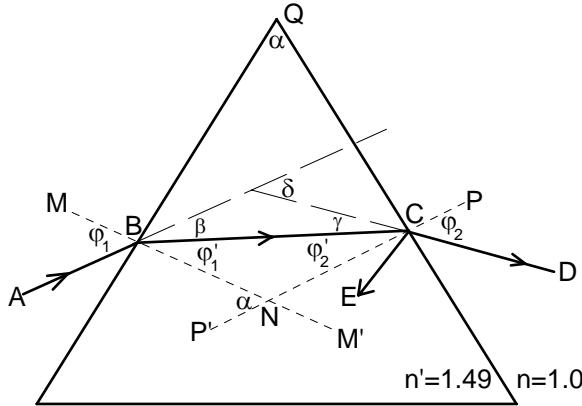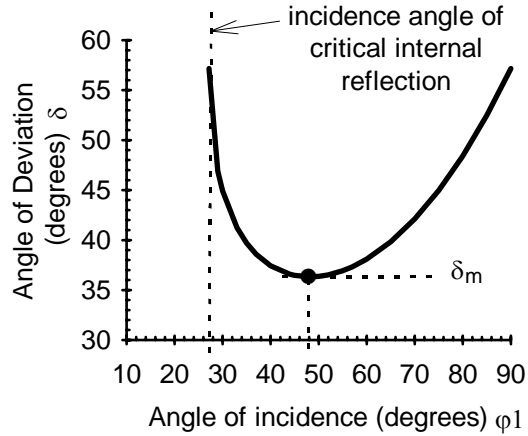Figure 3

Figure 4

Suppose that the laws of reflection and refraction are both followed by light travelling inside the prism. Let lines MM' and PP' be surface normals and let ray AB have incident angle φ_1. The angle of deviation of the refracted ray BC from the first surface is given by $\beta = \varphi_1 - \varphi_1'$. Ray BC is further deviated by angle $\gamma = \varphi_2 - \varphi_2'$ when it exits the prism at the second surface to give exit ray CD. The total **deviation** of ray CD from ray AB is

$$\delta = \beta + \gamma = \varphi_1 + \varphi_2 - \varphi_1' - \varphi_2'. \qquad (3)$$

Angle MNP' is α so triangle BCN implies

$$\alpha = \varphi_1' + \varphi_2', \qquad (4)$$

so

$$\delta = \varphi_1 + \varphi_2 - \alpha. \qquad (5)$$

Angle φ_2 is determined by the laws of refraction at the entrance and exit of the prism and α is a constant for a given prism. So Equation 5 implies that the angle of deviation for a given prism only depends on the incident angle of the ray and the index of refraction of the prism.

Figure 4 shows a graph of the expected deviation of an equilateral acrylic prism. The constants for this prism are α = 60° and n' = 1.49. As the incident angle decreases from 90°, the deviation decreases to a minimum value of 36.3°. This minimum is called the **angle of minimum deviation**, δ_m. At this angle, ray BC is parallel to the base of the prism. There is a relation between this minimum angle of deviation and the constants of the prism, which is

$$n' = \frac{\sin \frac{1}{2}(\alpha + \delta_m)}{\sin \frac{1}{2}\alpha}. \qquad (6)$$

As the incident angle decreases still further, the deviation begins to rise again to a maximum at an incident angle of 27.2°. At angles smaller than this incidence angle, no light is transmitted through the prism because because total internal reflection happens at point C. Ray CD vanishes and ray CE exits out of the bottom face of the prism instead.

Consider an object with refractive index *n*, having sides which are not planes. Although the laws of refraction and reflection still hold true, parallel rays incident on the surface, will not

remain parallel after interaction with the object as seen in fig. 5b. Similarily parallel rays will clearly not remain parallel after reflection from a curved mirror.
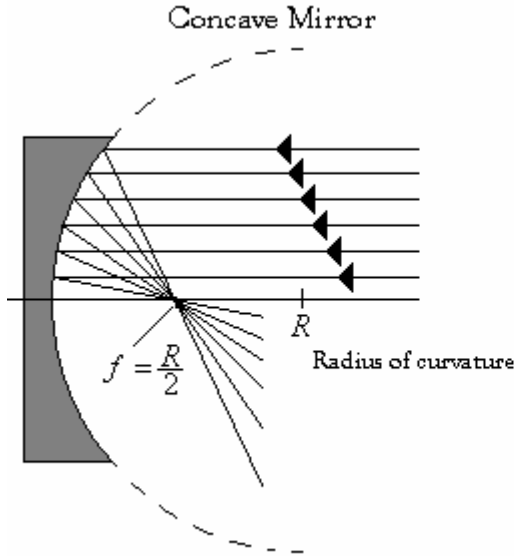
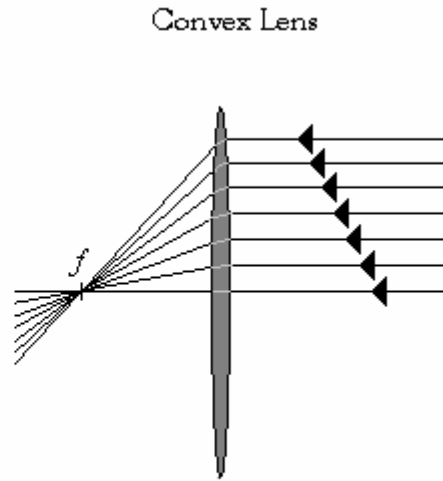Concave Mirror

Convex Lens

Figure 5(a)

Figure 5(b)

Consider special cases when the reflecting and refracting surfaces are spherical: a concave mirror and a convex lens. In both these cases, the parallel rays will be collected into a single point called the focal point (figure 5). The focal point of a concave mirror (figure 5a) is located at a distance

$$ f = \frac{R}{2} $$

(7)

away from the surface. The focal length of a convex lens is given by the **lensmaker's equation**

$$ \frac{1}{f} = (n_l - 1)(\frac{1}{R_1} - \frac{1}{R_2}) $$

(8)

Note that in this construction, the width of the lens is assumed to be negligibly small in comparison to the distance of object being viewed, and the image being formed:

It is important to remember here that using standard convention for a biconvex lens, $R_1 > 0$ and $R_1 < 0$ [otherwise (8) predicts that a symettric lens will have an infinite focal point.]

The radius of curvature for a given spherical lens may not be known in advance. Although there is a simple device called a diopter guage (used in the next experiment) that will give an accurate measure of R, a simple geometric construct is just as effective as displayed in figure 6. Marking a point halfway along the line segments *EF* and *GF*, and extending perpendiculars will yeild an intersection at the center of curvature *C*. The radius of curvature may then be measured as the distance between *C* and the optical surface:

$R = \overline{AC} = \overline{BC}$.

The experimental apparatus is shown in Figure 7. A cork board is placed on top of a stand for easier access and observation. To examine the law of reflection, a mirror is pinned to the cork board on top of a **mirror worksheet**. This worksheet is supplied with the experiment and extra copies are available as necessary. Examination of the mirror worksheet shows a protractor drawn beside a roughly triangular shape. The triangular shape is the mirror used for this experiment. Note that each side of the triangle is a
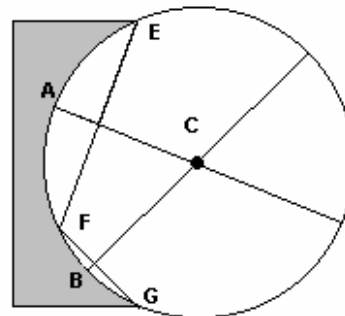

Figure 6

different type of mirror surface. One surface is a plane mirror, the other a concave mirror, and the third a convex mirror. Individual rays of light are traced by visually observing the alignment of 4 pins as shown in Figure 8. Two pins trace an incident ray of light and two others trace the corresponding reflected ray. The angle of incidence is selected by placing two pins on the protractor on the mirror worksheet at a particular angle. The reflected ray is found by placing two more pins that visually appear to line up with the reflection of the incident ray pins. The pins are aligned correctly when all four pins appear to line up. A ruler is then used to draw a line along each pair of pins. In this way an incident ray and reflected ray are drawn on the worksheet. The angle of the reflected ray can then be measured with a protractor. The law of reflection can be examined experimentally using this method.

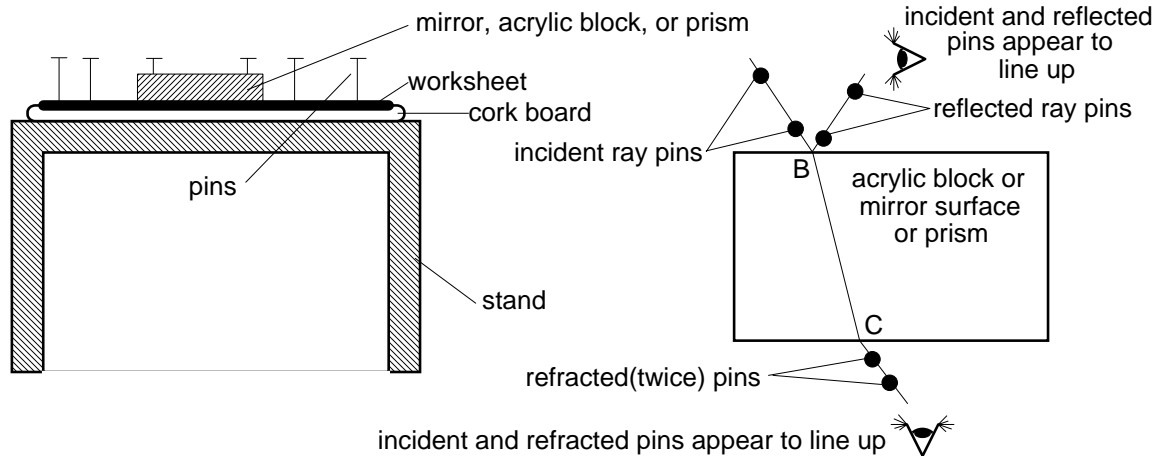Figure 7   Figure 8

Similarly, the rectangular plate worksheet and acrylic block can be pinned onto the cork board to examine the law of refraction. As with the mirror, two pins are placed at a given angle of incidence. Then, while sighting through the block, two more pins are placed so that all four pins visually appear to line up. The exiting ray can then be traced back to the exit point C with pencil and ruler. The worksheet has a dotted vertical line passing through the block. This line has two uses. First, the entrance point B is located on this dotted line at the top of the block. Connecting B to C with a line gives a tracing of the ray of light as it passes through the acrylic block. The angle of this line is measured with a protractor to determine the angle of refraction at point B. The second use of the dotted line is for lining up the protractor when measuring the angle of refraction.

The equilateral prism worksheet and acrylic prism can be pinned onto the cork board to investigate prism behaviour. Again, two pins can be placed at a particular angle of incidence on the protractor to create a ray going into the prism. The outgoing ray can then be investigated to measure the angle of deviation, find the angle of minimum deviation, or determine when critical internal reflection happens.

Lastly, the same procedure may be applied to the lens worksheet though it may be more difficult to secure the lens due to its shape. The pins can then trace incoming parallel rays and the outgoing rays can be traced as described above.

Experimental Procedure

1) The law of reflection can be experimentally examined by the behavior of a flat mirror. Since light travels in straight lines, an incident ray with a given angle of incidence can be simulated by placing two pins in a straight line at an angle to the flat mirror. Visually place two other pins so that they appear to line up with the reflection of the first two pins. These two pins represent the reflected ray. Use a straightedge to draw lines through the pins.

Put the supplied universal mirror onto the mirror worksheet and pin it into place. The protractor on the worksheet is used to set the angle of the two pins specifying the input ray. For at least four different incidence angles, plot the incident ray and the reflected ray. Measure the angle of reflection using a protractor. If the law of reflection is to hold, a graph of θ_i vs. θ_r should be linear with slope 1.

2) The angle of refraction can be examined experimentally by tracing rays of light through a rectangular plate of acrylic as in figure 2. Place the rectangular acrylic block onto the rectangular plate worksheet and pin it into place. The protractor on the worksheet is used to set the angle of two pins that represent the input ray AB. Once ray AB is in place, look through the rectangular plate in the direction from C to B. As your eyes move from side to side, the pins will be visible and at one point, both pins will appear to be in line with each other. At this point, place two more pins so that they line up with the first two pins. This is the exit ray CD. Use a straightedge to draw line representing the input and output rays. Light rays travel in a straight line inside the acrylic. So a straightedge can be used to draw a line connecting points B and C. This represents the path of ray BC. Use a protractor to measure the angle of refraction of ray BC. Measure the angle of refraction for at least six different angles of incidence.

3) Referring to figure 2, plot a graph of $\sin \varphi'$ vs. $\sin \varphi$ and determine the index of refraction n' from the slope of your graph.

4) The behavior of a light rays on a curved mirror can be explored just as in the case for a flat mirror. Using the mirror template, trace at least 6 separate incoming rays parallel to the mirror's optical axis and determine the point at which the reflected rays converge. This will give a value for the focal length f of the curved mirror. Using the geometric construction as in figure 6, determine the radius of curvature for the mirror and verify equation 7. Observe the image when an object (say, a pencil or a pen) is placed outside f, inside f, and at a distance of around f from the surface of the mirror. Explain these results.

5) Investigate the behavior of rays as they pass through an acrylic equilateral prism. The protractor on the prism worksheet is used to set the angle of incidence of the input ray. Visually lining up two pins on the exit side of the prism with the two pins of the input ray permit a ray of light to be traced as it passes through the prism. Trace the input rays for at least six incidence angles ranging from 35° to 65°. As in figure 3, extend input ray AB and exit ray CD with a straightedge, and measure the angle of deviation, δ, with a protractor.

6) Using the index of refraction for acrylic found in part three, rewrite equation 5 solely in terms of φ and n'. Plot a theoretical curve of δ as a function of φ and plot the values of δ measured in step five with error bars on top of the theoretical curve. Compare the experimental value for the angle of minimum deviation to that found by solving equation 6.

7) The thin lens equation 8 may be verified if both R_1, R_2, are known and the index of refraction of the lens has already been measured. In order to determine a value for the radii of curvature, draw a contour of the acrylic converging lens on paper and use the geometry of figure 6. Since a value of n has already been obtained for the material in step 2, f may be determined via equation 7. Using the converging lens template and pins provided, trace at least six rays parallel to the lens's optical axis through the lens and determine an average value for f. Compare to the value obtained for 7 and comment on your results.

Error Analysis

In this experiment one source of error that arises is the alignment of the pins. It is not possible to measure when the pins are all exactly aligned. However, it is possible to say with reasonable confidence that the pins are aligned **plus or minus half a degree**. The error in the pin alignment is then said to be half a degree. Once the pins are in place, other errors also arise. There is an error of half a degree in the procedure of drawing a straight line through the pins. There is also an error of half a degree in the accuracy of the angles as written on the graphical worksheets from distortions during the photocopying process. Furthermore, there will be some inaccuracy in the placement of the mirror and the acrylic rectangle and equilateral prism. Assume that this error is also half a degree.

Lastly, once the rays have been drawn, it is necessary to measure their angles with a protractor. Three sorts of errors happen during this measurement. First, the protractor must be lined up properly, in order to read off the angle. The error in doing this is called **the zero point error**. Secondly, there is some difference between the true angle and the markings of the angle on the protractor. This is called the **calibration error** of the protractor. Third, there is some uncertainty in reading the angle because the angle to be measured will not land exactly on a line of the protractor scale. This is called the **measuring error**. It has been found in practice that for instruments with scales (such as a protractor or ruler), the measuring error is often **half the smallest scale division** on the instrument. For the protractors used here, the smallest scale division is one degree, so the protractor measuring error is taken to be half a degree. It is estimated that each of the other two errors, the calibration error and zero point error, are at most half of a degree.

If we have a number of random errors Δx_i in an experiment, the total error is not simply the sum of the individual errors. The total error may be found via

$$\overline{\Delta x} = \sqrt{\sum_i (\Delta x_i)^2} \ . \qquad (9)$$

To be included in your Lab Report

1.   The mirror, block, prism, and lens worksheets with labeled rays.

2.   A plot of θ_i vs. θ_r .

3.   A plot of $\sin \varphi'$ vs. $\sin \varphi$ from step 3 along with the slope of the linear fit and a value for the index of refraction.

4.   A value for the radius of curvature and for the focal point of the curved mirror in step 4.

5.   Your geometric construction from step 4, as well as your description and explanation for the behaviour of the image of an object place inside, outside, and on the focal point of the curved mirror.

6.   The theretical curve plotted in step 6 as with the data points from step 5. Clearly label the angle of minimum deviation.

7.   Your calculation for the angle of minimum deviation from equation 6.

8.   Theoretical and measured values for f obtained in step 7.