Polarization Engineering

Equipment

2 Optical benches with pivot. Photodetector. Helium Neon Laser, 2 quarter-wave plates and 1 half-wave plate. 2 Linear Polarizers. Quartz plates.

Purpose

To understand the concept of polarization of light. To test Malus' law. To generate light of a given polarization and to determine the polarization of a given light.

Theory

Consider a plane electromagnetic wave propagating along the direction of the z axis. The electric field vector in this wave can be written as

$$\mathbf{E} = \mathrm{Re}\left[(E_{0x}\hat{i} + E_{0y}\hat{j})e^{i(kz-\omega t)}\right], \qquad (1)$$

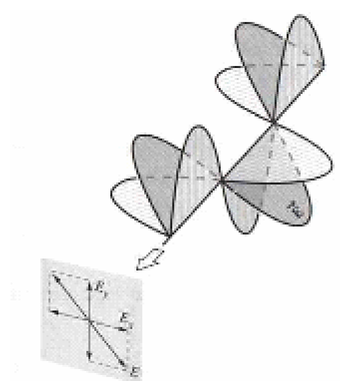

Figure 2

where $\hat{i}$ and $\hat{j}$ are unit vectors along the *x* and *y* axes, respectively; E_{0x} and E_{0y} are the complex amplitudes. The intensity of light in each polarization is proportional to

$$I_x = |E_{0x}|^2; \qquad (2)$$

$$I_y = |E_{0y}|^2. \qquad (3)$$

The total intensity is $I_{total} = |E_{0x}|^2 + |E_{0y}|^2$

Observing the trajectory of the tip of the electric field vector at a specific point in space, we find that, dependent on the amplitude and relative phase of the two component, the trajectory will form a certain pttern, which we call the **polarization pattern** or **polarization state**. A polarization state is **pure** when the amplitudes are constant in time. Pure polarization patterns of plane waves (1) are classified as follows:

- *Linear polarization*: $\arg E_{0y} = \arg E_{0x}(\pm\pi)$. There are important special cases:
  - *Horizontal polarization*: $E_{0y} = 0$
  - *Vertical polarization*: $E_{0y} = 0$
- *Circular polarization*: $\arg E_{0y} = \arg E_{0x} \pm \pi/2;\ |E_{0x}| = |E_{0y}|$
- *Elliptical polarization*: all other cases when the amplitudes are constant in time.

An example of various pure polarization states of light is shown in figure 5 which displays light such that $E_x = 2E_y$ for various values of $\varepsilon = \arg E_{0y} - \arg E_{0x}$. As we sweep the phase difference between the horizontal and vertical components, the polarization state evolves from linear to almost circular and back to linear.

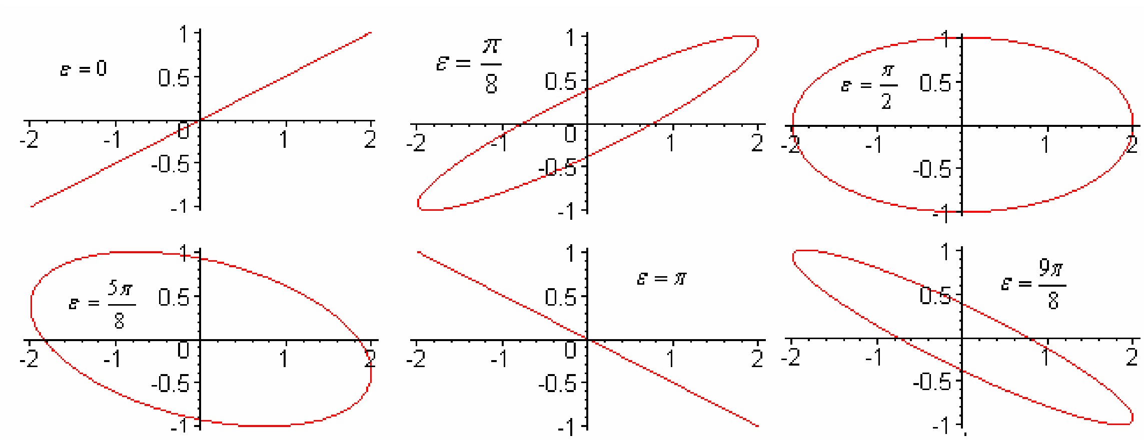

Figure 5

When the amplitude of the light wave changes in time[1], the polarization state is called mixed. For example, light from a light bulb or a flame is generally randomly polarized since the phase of a given component of light fluctuates randomly. Randomly polarized light is also called **natural** or **unpolarized**.

A **polarizer** is an optical instrument which removes the field polarization component perpendicular to a certain direction, the polarizer's transmission axis. If the reference frame is chosen such that the transmission axis is along the *x* axis, the wave (1) transmitted through the polarizer emerges as

$$\mathbf{E} = \hat{i}\,\mathrm{Re}\,E_{0x}e^{i(kz-\omega t)} \tag{4}$$

Consider a situation when a linearly polarized wave of amplitude E is incident on a linear polarizer with the transmission axis oriented at angle θ to the polarization of the wave. In the reference frame of the polarizer, we have

$$E_x = E\cos\theta\,;$$
$$E_y = E\sin\theta\,.$$

Only the *x*-component is transmitted through the polarizer. Since the intensity of the wave is proportional to the square of the amplitude of the electric field, the intensity transmitted by a linear polarizer which makes an angle θ with the transmission axis will be proportional to the square of the cosine of the angle. This is Malus' law:

$$I_{trans} = I_0\cos^2\theta\,. \tag{5}$$

We have seen that any polarized light may be represented as the superposition of horizontally (*x*) and vertical (*y*) polarized waves with the type of polarization depending of the phase differences between the two waves. In order to engineer a given polarization of light, it would then be beneficial to manipulate the phase difference in a predictable way. This is accomplished using **birefringent materials**, which retard the phase of one polarization component of the wave with respect to the other. A typical **wave plate** is a birefringent crystal cut so that the extraordinary axis (polarized parallel to the axis of anisotropy) is parallel to the

[1] Strictly speaking, a wave with mixed polarization cannot be called a plane wave. A plane wave must be completely monochromatic; its amplitude cannot change in time.

surfaces of the plate. When the extraordinary index of refraction is smaller than the ordinary, the extraordinary axis is called the **fast axis** and the ordinary axis is called the **slow axis**. Light polarized along the fast axis propagates faster than light polarized along the slow axis. Thus, depending on the thickness of the crystal, light with polarization components along both axes will emerge in a different polarization state (figure 6).

A **half-wave plate** introduces a relative phase shift of $\lambda/2$, or $2\pi/2=\pi$ radians. In the reference frame with the *y* axis oriented along one of the half-wave plate's axes, a plane wave (1) transmitted through the wave plate becomes

$$\mathbf{E}=\mathrm{Re}\left[(E_{0x}\hat{i}-E_{0y}\hat{j})e^{i(kz-\omega t)}\right]. \quad (6)$$

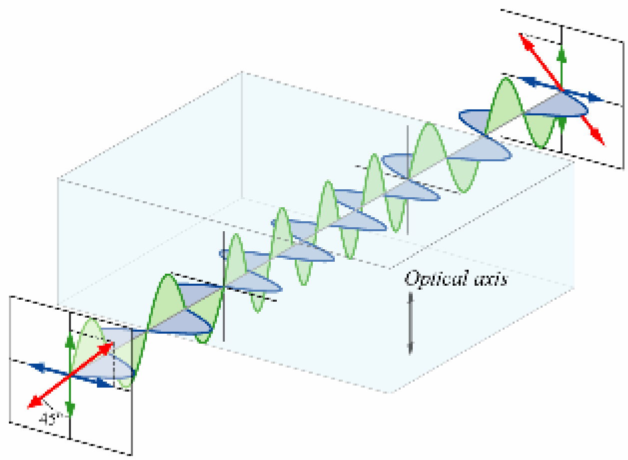

Figure 6

For example, when a half-wave plate is placed at an angle θ with respect to a incoming linearly polarized wave, the plane of polarization will be rotated by 2θ as in figure 7. For this reason, half-wave plates are sometimes called polarization rotators.

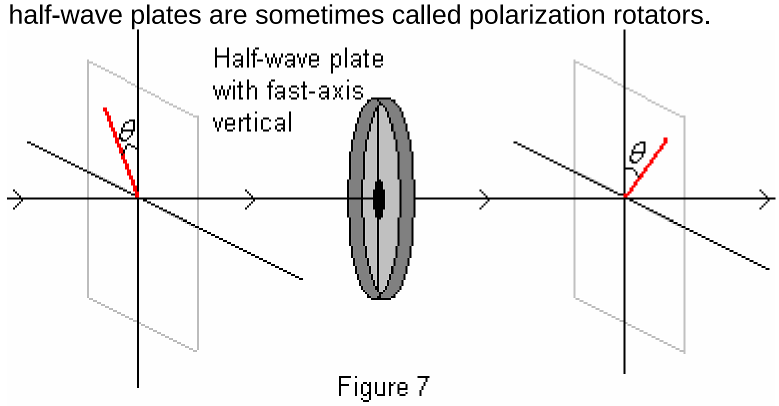

Figure 7

In a similar fashion, the **quarter-wave plate** retards a component of the polarization along the fast axis by $\pi/2$ radians. In analogy with equation (6), we write for a wave transmitted through a quarter-wave plate with its slow axis along the *y*-axis of the reference frame

$$\mathbf{E}=\mathrm{Re}\left[(E_{0x}\hat{i}+iE_{0y}\hat{j})e^{i(kz-\omega t)}\right] \quad (7)$$

When the plane of polarization is parallel to one of the axes, a waveplate has no effect on linearly polarized light, but when incident linearly polarized light is 45° to the quarter-wave plate, the one of the horizontal/vertical components of the polarized light receives the $\pi/2$ phase shift and circular polarized light emerges.

Certain materials possess a property called **optical activity**. When linearly polarized light is incident upon an optically active material, it emerges as linearly polarized light but with its polarization angle different from the original. The angle β of rotation of a wave that has propagated through an optically active material is proportional to the distance *L* of travel. The proportionality coefficient, usually measured in degrees/mm, is called the material's **specific rotation**. In this experiment, we study the optical activity of quartz.

A convenient mathematical description of the polarization state are the four Stokes parameters, which are sufficient to fully classify a polarization pattern. In fact, only three parameters are needed to characterize the polarization of light, since the first parameter merely states the total intensity of the light. Suppose that we measure the components of light that are polarized horizontally, vertically, +45°, -45°, right circular, and left circular which are denoted $E_H, E_V, E_+, E_-, E_R, E_L$ respectively. The Stokes parameters are then given by:

$$S_0 = \left\langle |E_H|^2 \right\rangle + \left\langle |E_V|^2 \right\rangle \tag{6i}$$

$$S_1 = \left\langle |E_H|^2 \right\rangle - \left\langle |E_V|^2 \right\rangle \tag{6ii}$$

$$S_2 = \left\langle |E_+|^2 \right\rangle - \left\langle |E_-|^2 \right\rangle \tag{6iii}$$

$$S_3 = \left\langle |E_R|^2 \right\rangle - \left\langle |E_L|^2 \right\rangle . \tag{6iv}$$

Note that the time-averaged magnitudes are simple the intensities of the given wave and that each Stokes parameter tells us something specific about it. S_0 gives the total intensity of the light and S_1 gives us the difference between the portions of the wave which are horizontally or vertically polarized – positive being horizontal and negative being vertical. Along the same lines S_2 tells us the difference between the components of light which are polarized at +45° or –45°, and S_3 gives us the difference of the circular components. Since the first Stokes parameter gives the total intensity, we must have that

$$S_0 \geq \sqrt{S_1^2 + S_2^2 + S_3^2} \tag{7}$$

and equality hold for completely polarized light. For partially polarized beams we have a strict inequality, the extreme being natural light which has $S_1 = S_2 = S_3 = 0$. Often Stokes parameters are grouped together as a vector, forming a **Stokes vector**. A general Stokes vector is given as

$$\vec{S} = \begin{pmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{pmatrix} . \tag{6}$$

The advantage of this form is that many optical elements may be then represented as operators in the form of a 4x4 matrix, known as a **Mueller matrix**. The Mueller matrices for some optical components are listed in table 1.

The resultant light emerging from an optical element is then calculated by a normal matrix multiplication. As an example suppose we send light in at +45° to a quarter wave plate with its fast axis horizontal. The emerging Stokes vector is then given by

Horizontal(+)/Vertical(-) Polarizer

$$\frac{1}{2}\begin{pmatrix} 1 & \pm 1 & 0 & 0 \\ \pm 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Polarizer at +/- 45^0

$$\frac{1}{2}\begin{pmatrix} 1 & 0 & \pm 1 & 0 \\ 0 & 0 & 0 & 0 \\ \pm 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Quarter-wave plate with fast axis vertical/horizontal

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \mp 1 \\ 0 & 0 & \pm 1 & 0 \end{pmatrix}$$

Half-wave plate with fast axis vertical

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Table 1

$$\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&-1\\0&0&1&0\end{pmatrix}\begin{pmatrix}1\\0\\1\\0\end{pmatrix}=\begin{pmatrix}1\\0\\0\\1\end{pmatrix}$$

which is seen to be right circular polarized light.

An alternative description of polarization is given by the **Jones vectors** and **Jones matrices**. A general Jones vector is given by:

$$\vec{E}=\frac{1}{\sqrt{\left|E_{0x}\right|^2+\left|E_{0y}\right|^2}}\begin{pmatrix}E_{0x}\\E_{0y}\end{pmatrix} \tag{8}$$

where φ_x, φ_y are the phases of the *x* and *y* components of the polarized light[2]. The Jones formalism has the advantage of having only two components and therefore a simpler representation, but the disadvantage of not being able to describe mixed polarization states. For example, horizontally polarized light has $E_{0y}=0$ so only the first component of the corresponding Jones vector is zero. Since the Jones vectors are normalized to have magnitude *1*, the Jones vector is then $\vec{E}_H=\begin{pmatrix}1 & 0\end{pmatrix}^T$. As a second example, recall that circularly polarized light has the y-component of light $\pi/2$ out of phase with the *x*-component, so that the Jones vector is:

$$\vec{E}_R=\begin{pmatrix}E_0e^{i\varphi}\\E_0e^{i\varphi-\pi/2}\end{pmatrix}=E_0e^{i\varphi}\begin{pmatrix}1\\e^{i\pi/2}\end{pmatrix}\to\vec{E}_R=\frac{1}{\sqrt{2}}\begin{pmatrix}1\\i\end{pmatrix}$$

where we've noted that for circular light, $E_{0x}=E_{0y}\equiv E_0$ and we've applied the normalization in the last step. In a similar fashion to the Mueller matrices, the Jones matrices are used to represent optical elements. Some Jones matrices are shown in table 2.

Horizontal Polarizer $\begin{pmatrix}1&0\\0&0\end{pmatrix}$ Vertical Polarizer $\begin{pmatrix}0&0\\0&1\end{pmatrix}$	Horizontal/Veriticle Quarter Wave-Plate $e^{i\pi/4}\begin{pmatrix}1&0\\0&\pm i\end{pmatrix}$	
Polarizer at +/- 45^0 $\frac{1}{2}\begin{pmatrix}1&\pm1\\\pm1&1\end{pmatrix}$	Rotation Matrix $R(\theta)$ $\begin{pmatrix}\cos\theta&-\sin\theta\\\sin\theta&\cos\theta\end{pmatrix}$	Rotates current reference frame by angle θ

Table 2

[2] Jones vectors are completely analogous to the way single-photon polarization states and states of a spin-1/2 particle are represented in quantum mechanics.

Procedure

The first step is to calibrate the equipment used in this experiment. The polarizers and wave plates will have a dial from 0° to 359°, but will not in general be configured such that the transmission axis or the fast axis corresponds to 0°. If however, we know how the instruments are calibrated with respect to one another, we can correct for any differences in between them.

1. First, select a polarizer P1 as the reference. Place P1 at 0° in front of the laser and place the other linear polarizer P2 between P1 and the detector[3]. Rotate P2 in the neighborhood of 90° determine the angle at which transmitted intensity is minimized. This corresponds to the 90° of P2.

2. Next, calibrate the quarter-wave plates: place P1 in front of the laser at 0°, and P2 in front of the detector at 90° (relative to P1 from step 1) so that there is no light transmitted. Next, place the first quarter-wave plate QWP1 between P1 and P2. Rotate QWP1 and notice now that there is light present of the detector except at the fast and slow axes of the QWP at which the incident linear polarized light is unaffected. Starting around 0° on QWP1 rotate until no light is present. This is one of the axes of the QWP. Repeat for the second quarter-wave plate QWP2 and for the half-wave plate HWP.

Although you have determined the positions of the waveplates' optical axes, you do not know which of the axes you found are slow, and which are fast. For the HWP, this makes no physical difference. For the QWPs, on the other hand, a confusion between the axes will result in a confusion between the right and left circular polarizations, which is physically significant. It is quite difficult to classify the axes you have found. However, it is possible to find out whether the axes you found for the two QWPs are of the same character.

If the two axes are of the same character (i.e. both fast or both slow), the quarter-wave plates superimposed with the axes collinear to each other will act as a single half-waveplate. If they are of the opposite character, the waveplate will compensate each other, so the transmitted light is unaffected.

To make a test, place P1 in front of the laser at 0°, then the two QWPs with the optical axes at 45°, then P2 in front of the detector at 90°. If the QWPs make up a half-wave plate, they will rotate the polarization by 90°, so almost the entire laser intensity will be transmitted through the second polarizer. If they compensate each other, the polarization will remain at 0°, and no light will be transmitted.

After this test, turn one of the QWPs by 90°. Now the combined waveplates should act oppositely compared to the previous setting. Record your findings.

Note that commercial waveplates never provide retardation precisely as specified, so do not be surprised if you do not obtain results exactly as expected.

3. Next perform a test of Malus' Law. Set the apparatus with P1 at 0° in front of the laser, and place P2 between the detector and P1. Starting at 0°, where the maximum light is incident on the detector, measure the intensity at the detector for the given angle of P2. Make at least 20 measurements from 0° to 180°.

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[3] It may be easier (and more precise) to determine the minimum transmission angle by observing the laser beam on a card rather than measuring it with the detector. You can decide which technique is better.

Measure the background light of the room by completely blocking the laser. Subtract this value from each of your measurements. Normalize your readings by dividing each measurement by the normalized maximum. Make a plot of normalized intensity as a function of polarizer angle. On the same graph, plot the theoretical curve given by equation (5).

4. Measure the Stokes parameters of the laser used in this lab. To this end, we must know I_H, I_V, I_+, I_-, I_R, and I_L. In order to measure I_H, place a linear polarizer in front of the detector and measure the intensity at 0° and 180°, averaging the results. Similarly, I_V is found by making measurements at 90° and 270°. Averaging the readings will give you a more accurate value and is not essential. I_+ and I_- are determined in a similar manner with the polarizer at $\pm$45°. In order to measure the left and right circular polarization intensities, set up a circular analyzer as in figure 8, with the quarter wave plate at 0° and the linear polarizer at $\pm$45° for I_R and I_L respectively. Using equations 6, calculate the Stokes parameters of the laser light. Verify equation 7, and give a brief description of the light produced by the laser.

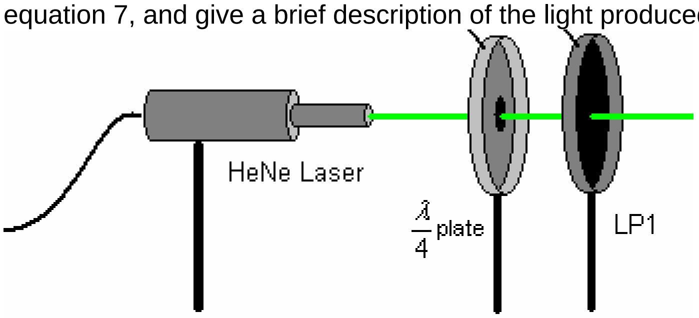

Figure 8

5. Produce light polarized at 0° with respect to P1. Insert the half-wave plate at 30° in between P1 and the detector. Measure the Stokes parameters of the resultant light.

6. Produce circularly polarized light as follows: place P1 at 45° in front of the laser followed by QWP1 at 0°. Measure the Stokes parameters of the resultant light.

7. Place the quartz rotator in the slide-holder. Sending in horizontally polarized light, measure the resultant Stokes parameters.

8. Given wave plates and polarizers, it is possible to engineer arbitrary polarization states. Here, you are asked to produce elliptically polarized light with the semi-major axis horizontal and the ratio to semi-major to semi-minor axes being 2:1. The basic idea of this is seen in figure 5. We first create linearly polarized light with $E_y / E_x = 1/2$ and then create a phase difference with a quarter wave-plate in a given component to produce elliptically polarized light.

Starting with horizontally polarized light, use a half-wave plate to rotate the plane of polarization to the correct angle (given by $\tan^{-1}(1/2)$ /2). Next place a quarter-wave plate with its fast axis horizontal in between the HWP and the detector.

Calculate the theoretical Jones vectors and Stokes parameters for the resultant light. Measure the Stokes parameters as in steps 4 – 7 and compare your measured result with your theoretically calculated values.

To be handed in with your report

1. (if this material has been covered in class) Verify that the matrices in Table 1 are correct.

2. A table giving the calibration details of your equipment along with the equipment number.

3. Your observations in step 2.

4. Your plot of Malus' law data with error bars obtained in step 3 with the theoretical curve on the same graph.

5. Explain why the setup in figure 8 makes a circular analyzer.

6. The Stokes parameters of the laser. What can be said about the polarization of the laser?

7. The Stokes parameters of the rotated light from step 5. Compare to your result in step 4.

8. A sketch of your setup in step 6, accompanied by the measured stokes parameters.

9. The resultant stokes vectors from step 7. How does the quartz affect the linearly polarized beam? Are your observations consistent with the accepted value for quartz at the He-Ne laser wavelength of about 19 degrees/mm? Is this dextrorotationary (clockwise) or levorotationary (counterclockwise) quartz?

10. A verbal description of how your polarization state was created in step 8. A calculation of the expected Jones vector for the resultant light. A calculation of the expected Stokes vector for the resultant light. The raw experimental data and the Stokes vector obtained in step 6. How does it compare with your calculation?