The Michelson Interferometer

Equipment

Pasco OS-8501 interferometer apparatus, Helium-Neon laser, laboratory stand with right angle bar clamp, Nalgene vacuum pump with air cell, 18mm focal length convex lens, 2 laboratory jacks, 30cm ruler, meter stick, wall mounted barometer, calipers.

Preparation

Study the interference of light and the history of the Michelson-Morley experiment.

Goals of the Experiment

To use and understand the Michelson interferometer. To use the interferometer to measure the wavelength of laser light. To use the interferometer to measure the index of refraction of air. To investigate how changes in pressure affect the index of refraction of air.

Theory

In 1887, Albert Michelson built his interferometer originally to investigate the existence of "ether", which was believed to permeate all space. It was the belief of many physicists at the time that ether was the medium through which light propagated, much like sound waves through air. The results of the famous Michelson-Morley experiment supported the idea that there is no stationary medium through which light propagates, which later formed the basis of Einstein's theory of relativity. The interferometer was later used to measure the wavelengths of atomic spectral lines with high precision, as well as displacements in terms of wavelengths of light. This enabled scientists to develop high precision length standards as well as improved methods for calibrating length measuring instruments. The interferometer can also be used to determine the **index of refraction** of transparent materials.

In this experiment, you will use a Michelson interferometer to determine the wavelength of laser light, as well as to investigate the index of refraction of air and how it is affected by changes in pressure. Figure 1 is a diagram of the apparatus, the Pasco model OS-8501 interferometer.
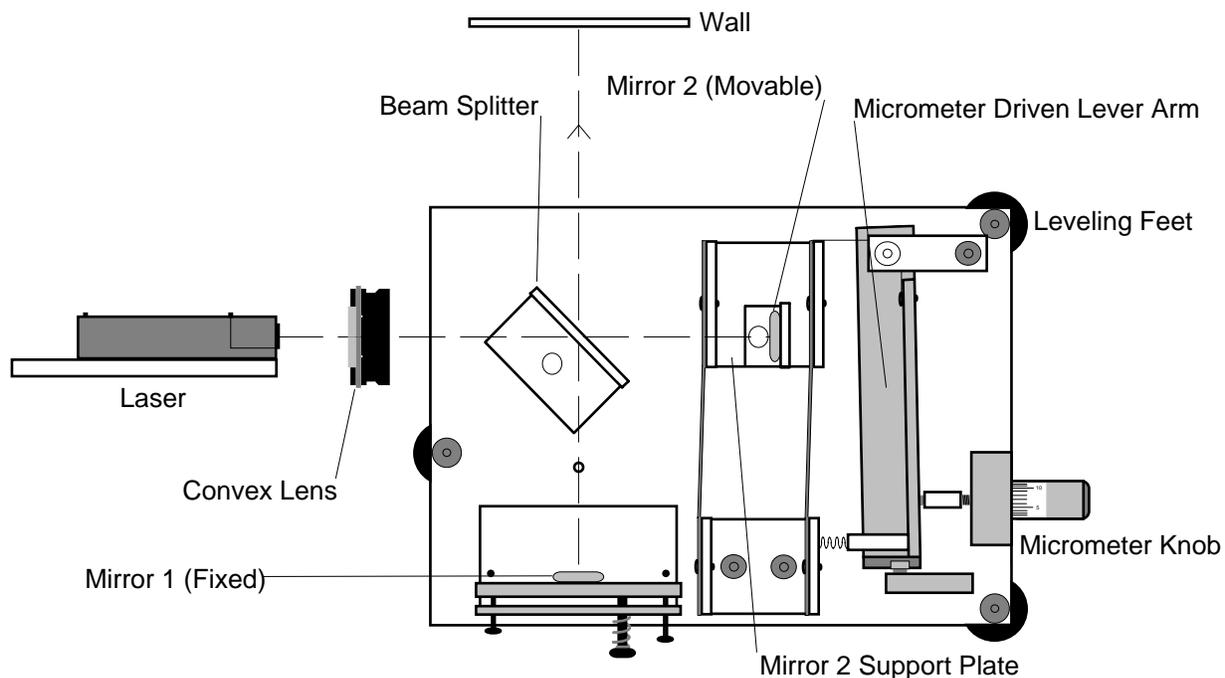

Figure 1

A simplified diagram of the Michelson interferometer is shown in Figure 2. Light from a monochromatic source passes through the beam splitter, producing two perpendicular beams of equal intensity. The two beams then reflect off of two separate mirrors which are deliberately located at different distances from the beam splitter. The mirrors are aligned in such a way that the beam is reflected straight back along the incoming path. When the beams recombine at the beam splitter, they will **interfere** with each other. Whether the interference will be **constructive** or **destructive** depends on the relative phase of each of the combining light beams. This is determined by the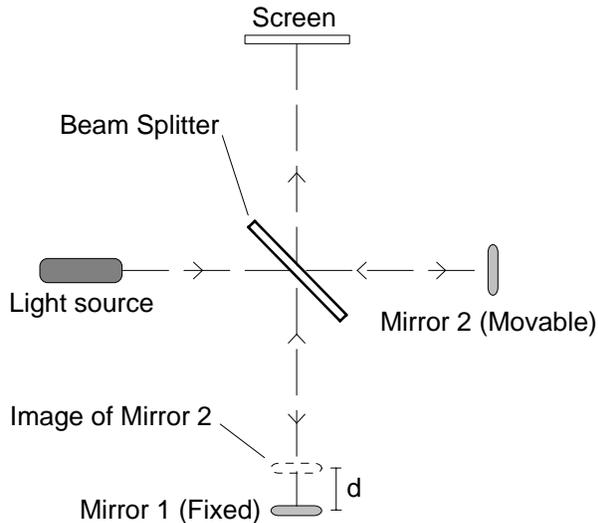 path length difference, 2d. With constructive interference, the wave amplitudes add in such a way to produce a maximum intensity beam striking the screen. The condition for maximum constructive interference is

$$2d = m\lambda,\qquad(1)$$

where m is an integer and λ is the wavelength of the incoming light. When the path length difference is an integer multiple of the wavelength, the recombining light beams will be in phase since both light beams originated from the same source. The resulting amplitude of the combined beam is then the sum of the amplitudes of each beam.

With destructive interference, the phases of the light beams are such that the recombining beams cancel each other out. The condition for maximum destructive interference is

$$2d = \left(m+\frac{1}{2}\right)\lambda.\qquad(2)$$

Figure 2

Figure 4

When the path length is an odd half integer multiple of the wavelength, the recombining light beams will be exactly out of phase. The resulting amplitude of the combined beam will be the difference of the amplitudes of each beam. Moreover, since the amplitudes of the split beams are equal, the combined light beam will have zero amplitude.

By moving one of the mirrors, we can change the path length difference and the relative phase of the light beams. With careful alignment, it is possible to use a laser light source to produce interference. As the path length difference changes, we would see both constructive and destructive interference. By moving Mirror 2 slowly towards the light source, we would see the laser point on screen appear, reach maximum brightness, fade away, and then disappear as the path length difference is increased by one wavelength.
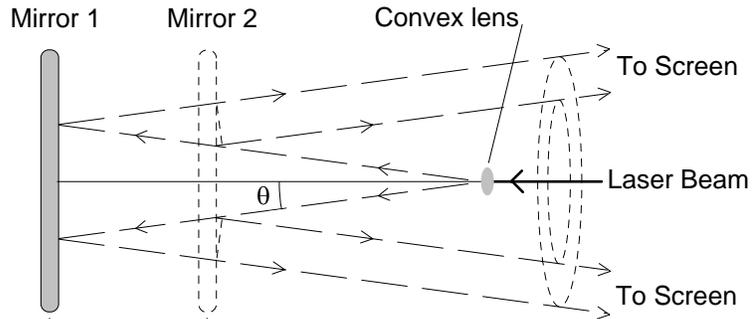
Figure 3

It is more practical to use a dispersed beam instead of a thin laser beam. In the lab, you will use a convex lens to disperse a laser light source. With a dispersed beam, the interferometer produces an interference pattern on the screen instead of a single point. Figure 3 shows the path of a dispersed laser beam at an exaggerated angle. For convenience, the primary elements of the interferometer are shown in a linear arrangement. The parallel beams reflected towards the screen interact with each other in a constructive or destructive manner, depending on the path length difference. The path length difference, now dependent on the beam angle, θ, is $2d\cos\theta$. Since the path difference is dependent on the angle of the beam there will be certain angles where there is constructive interference and certain angles at which there is destructive interference.

The condition for constructive interference is now

$$2d\cos\theta = m\lambda \ . \hspace{4cm} (3)$$

Constructive interference at specific angles gives rise to an interference pattern of bright and dark concentric circular **fringes**, as in Figure 4. Note that Equation 1 still holds as a special case where $\theta=0$, corresponding to the center of the interference pattern.

With a dispersed beam, we can change the path length difference by moving Mirror 2 and we would see changes in the interference pattern. The interference fringes will move either towards or away from the center of the interference pattern, fringes are either produced or annihilated. In the center of the interference pattern, we would see the alternating light and dark spot as discussed before. A fringe will be produced or annihilated for every wavelength by which the path length difference is changed. For this reason, very fine control is required to move the mirror. The micrometer screw (see Figure 1) is designed to control and measure the small displacements of Mirror 2. Mirror 2 is restricted to a path parallel to that of the laser beam. As a result, the distance traveled by Mirror 2 is equal to the change in mirror separation, Δd. By counting the number of passing fringes corresponding to a measured mirror displacement, we can calculate the wavelength of the laser light. The number of fringes that have passed in a mirror displacement, Δd, is equal to the number of wavelengths that the path length difference has changed. Therefore, Equation 1 implies the wavelength is given by

$$\lambda = \frac{2\Delta d}{N} , \hspace{4cm} (4)$$

where N is the number of passing fringes corresponding to the change in mirror separation.

We can further investigate the interference pattern in order to better understand it's origin. We can measure the fringe spacing of the stationary interference pattern, then compare with predictions made by the theory. Given a constant path difference and constant wavelength, there are values of θ and m that satisfy Equation 3, which correspond to each fringe. So we have

$$2d\cos\theta_N = m_N\lambda , \hspace{4cm} (5)$$

where θ_N is the angle of the Nth fringe from the center of the interference pattern, and m_N is the integer number of wavelengths associated with the Nth fringe path difference. If we adjust d so that there is a fringe of maximum brightness at $\theta=0$, then m_0 is the value of m that satisfies Equation 3 for $\theta=0$. So then

$$2d\cos(0) = 2d = m_0\lambda , \hspace{4cm} (6)$$

and m_0 is the integer number of wavelengths equal to twice the mirror separation (note that this is a re-statement of Equation 1, the zero angle condition for constructive interference). Since neighboring fringes differ in path length difference by one wavelength,

$$m_0 - m_N = N \ . \hspace{4cm} (7)$$

Equations 5, 6, and 7 together predict that the cosine of the angle between the Nth fringe and a fringe at the center of the interference pattern will be given by

$$\cos\theta_N = 1 - \frac{N\lambda}{2d} . \hspace{4cm} (8)$$

By plotting the cosine of the fringe angle versus the fringe number, we can see how well the interference pattern matches the one predicted by the theory.

The interference pattern is sensitive to changes in the relative phase of the two split beams. So we can use the interferometer to investigate how transparent objects affect the phase of light by placing them in one of the split beam paths. The laboratory apparatus includes an air cell connected to a vacuum system. The air cell is an air tight cylinder with glass windows on both faces. We can use this air cell to investigate the index of refraction of the air inside. Consider an air cell placed in the path of one of the split beams, as shown in Figure 5.

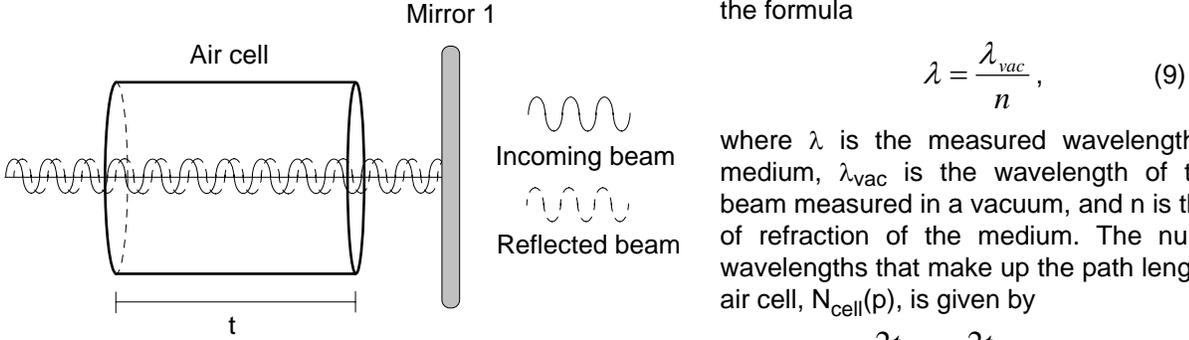As a light beam passes though a medium, the wavelength of light is dependent on the index of refraction by the formula

Mirror 1

Air cell

Incoming beam

Reflected beam

$$\lambda = \frac{\lambda_{vac}}{n}, \qquad (9)$$

where λ is the measured wavelength in the medium, λ_{vac} is the wavelength of the light beam measured in a vacuum, and n is the index of refraction of the medium. The number of wavelengths that make up the path length in the air cell, $N_{cell}(p)$, is given by

t

Figure 5

$$N_{cell}(p) = \frac{2t}{\lambda_{cell}} = \frac{2t}{\lambda_{vac}}n(p), \qquad (10)$$

where t is the thickness of the cell, λ_{cell} is the wavelength of the light in the cell, p is the pressure inside the cell, and n(p) is the index of refraction of air at pressure p. Since the index of refraction is a function of pressure, so is the number of wavelengths in the air cell.

It is important to note that the index of refraction of a material medium is always greater than 1, and the index of refraction of a vacuum is equal to 1. This means that the wavelength (as well as the speed) of a beam of light is a maximum in vacuum conditions. As air is evacuated from the cell, the wavelength of the laser light in the cell will increase as the index of refraction approaches 1. For each wavelength decrease in the cell, the relative phase of the split beams will have undergone one complete cycle. This means that the fringes of the interference pattern will move as the air is evacuated from the cell. The number of passing fringes is equal to the change in number of wavelengths in the air cell. From this we know that the number of passing fringes as the air cell pressure changes from atmospheric pressure to some pressure, p, is equal to the difference in number of wavelengths in the air cell at the two pressures. With Equation 10, we have

$$N_{diff}(p) = N_{cell}(p_{atm}) - N_{cell}(p) = \frac{2t}{\lambda_{vac}}\left[n(p_{atm}) - n(p)\right], \qquad (11)$$

where p_{atm} is atmospheric pressure, and $N_{diff}(p)$ is the number of passing fringes as the air cell is evacuated from atmospheric pressure to some pressure p. Re-arranging Equation 11 with Equation 9, we have

$$1 - \frac{\lambda N_{diff}(p)}{2t} = \frac{n(p)}{n(p_{atm})}, \qquad (12)$$

where λ is the wavelength of the laser light in air at atmospheric pressure.

Note that the quantities on the left side of Equation 12 and the pressure inside the air cell are measurable. We can evacuate the air cell while counting the number of passing fringes and recording the pressure. From this data, we can then calculate the relative index of refraction, $n(p)/n(p_{atm})$, at each of the pressures. This makes it possible to observe how the index of refraction of air is affected by changes in pressure. We can also determine the index of refraction of air at atmospheric pressure from a plot of $n(p)/n(p_{atm})$ versus p. The vertical intercept of this plot yields a value for $1/n(p_{atm})$ since n(p)=1 at p=0. However, $n(p)/n(p_{atm})$ is always 0.999... so it works better to plot $1-n(p)/n(p_{atm})$. Then the intercept at zero pressure is $1-1/n(p_{atm})$ instead.

Experimental Procedure

1. Begin by aligning the interferometer. Place the interferometer on one of the laboratory jacks about one meter from the wall (we will use the wall as a screen to view the interference pattern). The laser should be attached to the laboratory stand with the right angle bar clamp. Place the laser about 30cm from the interferometer as shown in Figure 1. Loosen the thumb screw that holds the beam splitter and rotate it so that it is out of the path of the beam. Although the angle of Mirror 2 is adjustable, the mirror should be flush with the Mirror 2 support plate and facing the laser as shown in Figure 1. Now align the laser so that the beam reflects off the center of Mirror 2 directly back towards the aperture of the laser. Rotate the beam splitter into the laser path so that it is at an angle of approximately 45° to the beam. You should see two laser spots on the wall. Rotate the beam splitter until the spots are as close as possible, then tighten the thumb screw. Use the alignment screws on Mirror 1 to superimpose the two spots on the wall.

Now we will diverge the beam in order to produce the interference pattern. Place the convex lens on the second lab jack in between the interferometer and the laser as shown in Figure 1. A second lab jack is necessary to keep the orientation of the interferometer undisturbed. For best results, the lens should be placed approximately 4cm from the interferometer. Adjust the orientation of the lens until you see the dispersed beam reflecting off the center of Mirror 2. You should now see an interference pattern on the wall. You may need to make fine adjustments using the alignment screws on Mirror 1 in order to center the concentric circle pattern. When the alignment is complete, you should see an image on the wall like the one in Figure 4.

2. Adjust the micrometer knob so that the micrometer driven lever arm is approximately parallel with the interferometer base. Turn the micrometer knob about one full turn counterclockwise until the zero on the knob is aligned with the reference mark. You should see moving fringes as the micrometer knob is adjusted.

3. You will now use the micrometer to measure the wavelength of the laser light. Turn the micrometer screw slowly counter clockwise while counting the number of passing fringes. After about 80 fringes have passed, record the number of fringes as well the mirror displacement from the micrometer screw. Turn the micrometer screw back to the original position and repeat to obtain two more measurements.

4. Adjust the micrometer screw so that the center of the interference pattern is an illuminated dot of maximum brightness. This implies that there is constructive interference at $\theta=0°$ and Equation 6 is satisfied. Measure the radius of each fringe from the center of the interference pattern using the ruler. Record the radius and the corresponding fringe number. Measure the distance from Mirror 1 to the wall with a meter stick. From the fringe radius and the distance to the wall, one can determine the angle of the fringe, θ, in order to compare with the predictions made by Equation 8.

5. Record the air pressure with a barometer.

6. Place the air cell in the beam path between the beam splitter and Mirror 1. Be sure that the air cell is parallel with the beam path throughout the entire procedure step. Press the pressure release button on the Nalgene pump in order to be sure that the initial air cell pressure is equal to atmospheric pressure. Record the initial pressure reading on the Nalgene pump gauge. Slowly squeeze the pump until the fringe interference pattern has completed one cycle, and record the pressure. Continue to evacuate the air and record the air cell pressure as well as the number of cycles after each passing fringe. Repeat this until the pressure reaches the minimum value and the pump can no longer evacuate air from the cell. When you are finished, be sure to press the pressure release button on the pump. It is important to note that the pressure gauge reads "mmHg vacuum", therefore, to find the absolute pressure inside the air cell, you must subtract the difference between the pressure on the gauge and the initial pressure from the air pressure read on the barometer.

Error Analysis

There will be measurement error in the mirror displacement which can be taken to be one half the smallest division on the micrometer screw. When counting large numbers of passing fringes, even though you are counting whole numbers, it is a good idea to use an uncertainty of one fringe since there is freedom to move the micrometer without a complete fringe passing. When measuring length with a ruler, the error is usually taken to be one half of the smallest division. The error in pressure can also be taken as one half the smallest division on the pressure gauge.

Systematic errors arise because the apparatus can't be perfectly aligned. It is possible to observe an interference pattern even if the laser is slightly misaligned, however, the instrument will deliver poorer results since the mirror movement mechanism is not calibrated for such a situation. If the alignment procedure is followed, the apparatus should function with reasonable accuracy.

To be handed in to the Laboratory Instructor

Prelab

1. A derivation of the path length difference of a dispersed laser beam in terms of the beam angle. Show that the path length difference between paths ABE and ACD is given by 2dcosθ. Note the dotted line, indicating that the lengths of DM2 and EB are unequal.
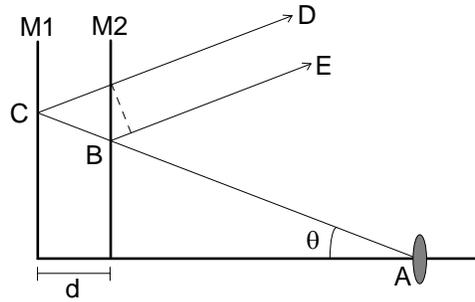
Data Requirements

2. A table of values for the number of passing fringes, N, and the corresponding change in mirror separation, Δd, with errors. Include a column of the calculated wavelength with error.

3. A table of values for the fringe number, N, with the corresponding fringe radius as measured in procedure step 4, with errors. From the fringe radius and the wall distance, calculate the $\cos\theta$ value for each of the fringes, with errors. For this calculation you can assume the beam is perpendicular to the wall and that the beam does not strike the mirrors far from the center.

4. A plot of $(1-\cos\theta)$ versus N, with error bars. Is it a linear relationship with reasonable slope and intercept?

5. A table of values for the number of passed fringes, $N_{diff}(p)$, and the corresponding pressure, p, as measured in procedure step 6, with errors. Include a column of the calculated $n(p)/n(p_{atm})$ values. In the calculation, you should use the average of the three wavelength values that you measured in procedure step 3 as the wavelength of the laser light at atmospheric pressure. Also note that the thickness of the air cell is 3.0cm.

6. A plot of $(1-n(p)/n(p_{atm}))$ versus p with error bars. Extrapolate the plot to find the $1-n(p)/n(p_{atm})$ value at zero pressure. From the intercept, obtain the index of refraction of air at atmospheric pressure.

Discussion

8. Compare your wavelength with the expected wavelength (the expected value of the laser light wavelength is 632.816nm) using data from 2.

9. Comment on whether or not the fringe spacing is consistent with Equation 8.

10. Compare your measured value of the index of refraction of air with the accepted value of 1.00025. Comment on the observed relationship between the index of refraction of air and pressure.