Fizeau Bands

Equipment

Large and small Newton's rings apparatus, air wedge apparatus, lab jack, traveling microscope, sodium lamp, riser block, aluminum foil, beam splitter, beam splitter stand, micrometer, lens paper, lens cleaner, interference color chart, thin cover glass, diffuser.

Purpose

To investigate the interference phenomena resulting from multiple reflections in thin films. To apply this theory to the measurement of the radius of curvature of a lens and the thickness of a thin object such as a hair.

Theory

Interference of light is a commonly observed phenomenon. It can be seen in the intricate color patterns produced by thin films of oil on water, by soap bubbles, or by cracks in a piece of glass. These color patterns are caused by the multiple reflection of light between two surfaces of a thin film of transparent material.

Consider the reflection of light from a plane parallel film as shown in Figure 1. When a ray of light from a source S is incident on the surface of the film at point A, the ray splits. Part of the ray is reflected as ray 1 and part is refracted in the direction AF. When the refracted ray reaches F, it will again be split. Part of the refracted ray will be reflected to B and part refracted towards H. At B the ray will again be divided, repeating this process throughout the film. The intensity decreases rapidly from one ray to the next. This process results in two sets of parallel rays, one on each side of the film. If one set of parallel reflected rays is collected by a lens L, and focused at a point P, each ray will have traveled a different distance. Because of the phase relations, constructive or destructive interference will be produced at the point P. This is the effect observed when colors of thin films are observed with the naked eye. In that case L is the lens of the eye, and P lies on the retina.

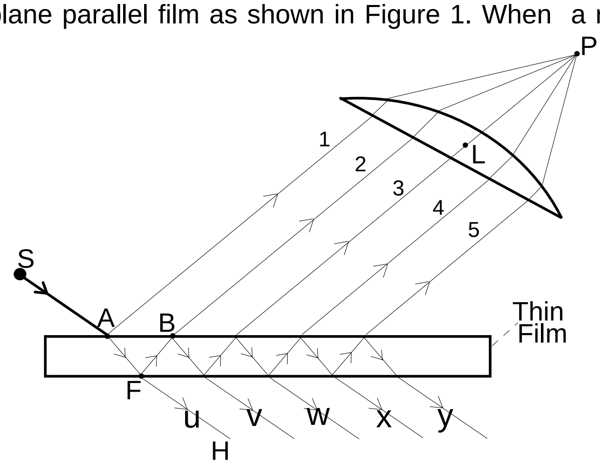

Figure 1

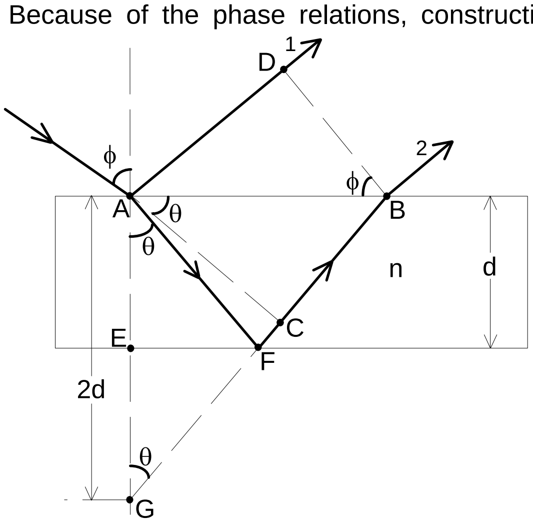

Figure 2

In order to find the phase difference between the rays, we must first find the difference in the optical path traversed by a pair of successive rays, such as rays 1 and 2. A more detailed diagram of these two rays is shown in Figure 2. Let d be the thickness of the film, n its index of refraction, λ the wavelength of the light, ϕ the angle of incidence, and θ the angle of refraction. If BD is perpendicular to ray 1, the optical paths from D and B to the focus of the lens will be equal. Starting at A, ray 2 has the path AFB in the film and ray 1 the path AD in air. The difference, Δ, in these optical paths is given by

$$\Delta = n \cdot AFB - AD \,, \tag{1}$$

where the index of refraction of air is essentially 1.0. If BF is extended to intersect the perpendicular line AE at G, then

$$AF = GF\ , \tag{2}$$

because of the equality of the angles of incidence and reflection at the lower surface. Thus we have

$$\Delta = n \cdot GB - AD\,. \tag{3}$$

Since

$$GB = GC + CB\,, \tag{4}$$

the difference in optical paths can be then be written as

$$\Delta = n \cdot (GC + CB) - AD\,. \tag{5}$$

Because AC is drawn perpendicular to FB, the dotted lines AC and DB represent two successive positions of a wave front reflected from the lower surface. The optical paths must be the same for any ray drawn between two wave fronts. Therefore we may write

$$n \cdot CB = AD\,. \tag{6}$$

The path difference then reduces to

$$\Delta = n \cdot GC\,. \tag{7}$$

Expressing GC in terms of d and θ gives us

$$\Delta = n2d \cos\theta\,. \tag{8}$$

If the path difference is a whole number of wavelengths, you might expect rays 1 and 2 to arrive at the focus of the lens in phase with each other and produce a maximum of intensity. However, we must take into account the fact that ray 1 undergoes a phase change of π at reflection, while ray 2 does not, since it is internally reflected. The condition

$$m\lambda = 2nd \cos\theta\,,\quad m = 0,1,2,\ldots\,, \tag{9}$$

then becomes the condition for maximum **destructive interference**, where m is the order of interference, and λ is the wavelength of light. Maximum **constructive interference** will then occur at half wavelength path differences, such that

$$\left(m + \frac{1}{2}\right)\lambda = 2nd \cos\theta\,,\quad m = 0,1,2,\ldots\,. \tag{10}$$

Since the human eye is more sensitive to small changes of low intensity of light rather than to small changes of high intensity of light, the dark bands of maximum destructive interference are normally used to make measurements.

In the above derivation, only the interaction of the first two rays was discussed. Since intensity of the ray decreases each time it is divided into a reflected and refracted ray, the first two rays will have the highest intensity, and hence will have the most impact on the interference pattern. This is not to say that the other rays do not affect the interference pattern. On the contrary, the reason complete darkness is observed at maximum destructive interference is because rays 3,4,5,.., are all in phase with ray 2. The addition of all these rays results in an intensity as great as that of ray 1, allowing for complete cancellation. Although inclusion of other rays does affect the magnitude of the interference, they do not change the conditions for maximum constructive or destructive interference.

Names are given to the different types of interference bands that result from varying different factors in Equations 9 and 10. If the right hand side of the equation is kept constant and the wavelength is allowed to vary, then for certain wavelengths destructive interference results, whereas for others, constructive interference results, depending on the value of m. These interference bands are known as **fringes of equal chromatic order**, or **FECO bands**. If on the other hand, wavelength is fixed, and the angle of refraction varies, the resulting interference bands are called **Haidinger bands**, or **fringes of equal inclination**. The last type of interference bands, called **Fizeau bands**, are the result of varying d. The distinction between the three types of bands is not absolute though, for more than one factor may vary at the same time, and the types become merged.

The type of interference we wish to investigate here are Fizeau bands in thin films of air. Since we are really concerned with the effects of d on the band pattern, a simplified version of Equation 9 can be applied. If the thin transparent film used is air, then the index of refraction will be very close to 1.0. Also, if the angle of incidence is 0º, then the condition for destructive interference becomes

$$m\lambda = 2d\ . \qquad (11)$$

When monochromatic light is used to observe Fizeau bands, they are interpreted in exactly the same manner as the contour lines on a topographical map. The larger the thickness of the film, d, the larger the spacing between the bands. This relationship though, deals with height differences on the order of micrometers, as opposed to height differences on the order of hundreds of meters for topographical maps. Because the distance between Fizeau bands is dependent on the thickness of a thin film, they have many practical uses. They are , for example, used for testing the flatness of optical surfaces. An object is tested by laying it on top of a known optical flat and observing the resultant band pattern. If the bands appear parallel, straight, and equally spaced, then the tested surface is flat.

Fizeau bands were actually one of the first types of interference bands to be observed. They were first seen by Robert Boyle in 1663. He observed them as concentric rings centered around the contact between a long radius convex lens and a flat glass plate, as demonstrated in Figure 3. Because Newton studied these interference bands further, they are now known as **Newton's rings**.

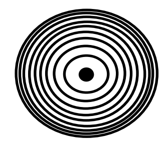

Figure 3

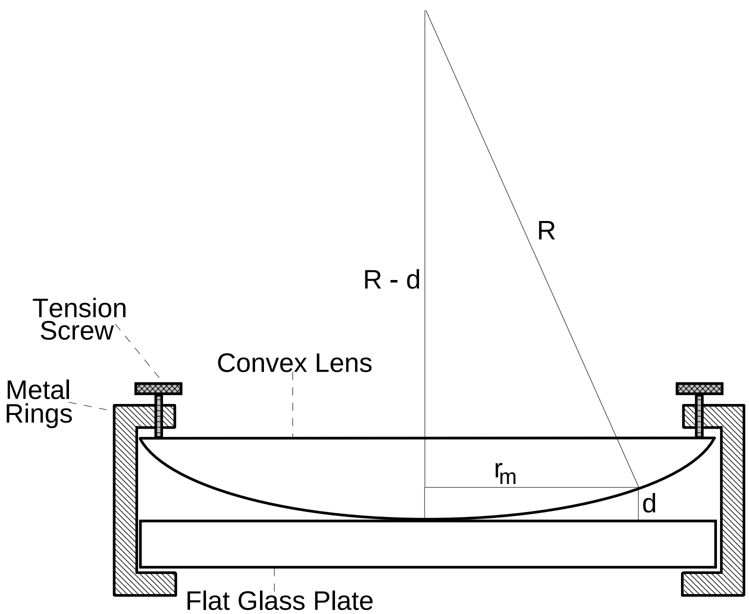

Figure 4

Newton's rings are produced by the air gap between a flat glass plate and a long radius convex lens. An apparatus used to produce such an interference pattern is shown in Figure 4. The curvature of the convex lens is exaggerated in order to demonstrate the desired effect. The lens, in reality, appears, to the naked eye, to be flat. If the convex lens did have a large curvature, the rings would be so close together we would not be able to see them. Since the spacing between the rings decreases as the air gap increases, the ring pattern observed should get closer together as we move outward from the point of contact between the lens and the glass plate. By measuring the change in spacing we can determine the curvature of the convex lens. To see how this can be done, consider again Figure 4. In this figure, R represents the radius of curvature of the convex lens, r_m, the distance from the center to the mth ring, and d, the air gap between the lens and the flat glass plate at the mth ring. By applying the Pythagorean theorem we get

$$R^2 = (R-d)^2 + r_m{}^2\ . \qquad (12)$$

By expanding this out and collecting like terms, the equation becomes

$$r_m{}^2 = 2Rd(1-d)\ . \qquad (13)$$

Since d is much smaller than 1, this can be simplified to get

$$r_m{}^2 = 2Rd\ . \qquad (14)$$

Combing Equations 11 and 14, gives a relationship for R in terms of the distance to the mth ring and the wavelength of light used, namely,

$$R = \frac{r_m^{\ 2}}{m\lambda}. \tag{15}$$

Fizeau bands can also be used to determine the thickness of very thin objects like hair, paper, or aluminum foil. The thickness of thin objects can be determined through the use of an **air wedge**, as demonstrated in Figure 5. The air wedge works on the same principle as Newton's rings. The larger the air gap, the smaller the separation between the lines. As long as the glass surfaces are perfectly flat, the bands will be straight, otherwise they will tend to curve, forming contour lines from the unevenness in the surfaces.

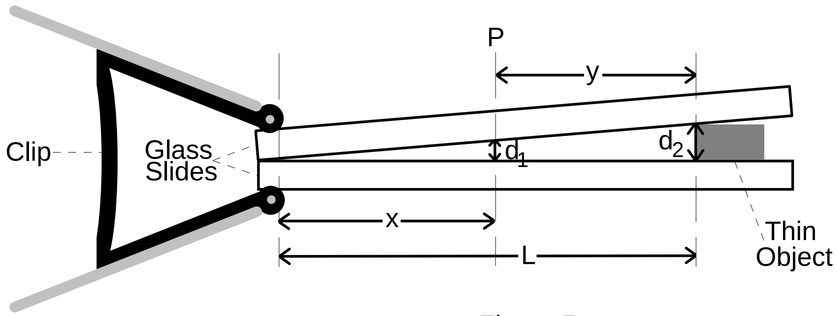

Figure 5

Suppose that the interference bands are only due to the formation of an air gap caused by the placement of an thin object between two glass slides. We can derive an equation which will give us the thickness of the object in terms of the distance between the object and the edge of the clip, L, and the distance, y, from the object to a point P. From similar triangles we find that

$$\frac{d_2}{d_1} = \frac{m_2}{m_1} = \frac{L}{x}, \tag{16}$$

where m_2 is the number of dark bands from the edge of the object to the edge of the clip, and m_1 is the number of dark bands from any point P to the edge of the clip. Because of distortions caused by using a clip to hold the two glass slides together, it is hard to get an accurate value for m_1. We can avoid this problem by counting the bands from the edge of the object to some point P, so that

$$m = m_2 - m_1. \tag{17}$$

We then use the corresponding distance y by applying the relation

$$y = L - x. \tag{18}$$

The thickness of the object, d_2, can then be determined by combining Equations 11, 16, 17, and 18, so that

$$d_2 = \frac{m\lambda}{2}\frac{L}{y}. \tag{19}$$

Even though interference bands are visible to the naked eye, their spacing is such that you need a microscope in order to make an accurate measurement. The experimental setup used to measure the spacing between Fizeau bands is shown in Figure 6. A monochromatic light source, in this case a Sodium lamp, illuminates the arrangement thereby keeping λ constant. A traveling microscope is used to count the number of bands and measure the related distances. A beam splitter allows the light to be incident on the Newton's rings or air wedge apparatus surface at nearly 0^o, while allowing us to view the surface at the same angle. This lets us avoid using the cosine term, allowing the use of Equation 11. The microscope sits on the beam splitter stand which is placed on a lab jack so that the beam splitter is at the same height as the light source. The Newton's rings or air wedge apparatus is placed under the beam splitter on a riser block. The riser block is put in place in order to keep the bands within the focusing range of the microscope.

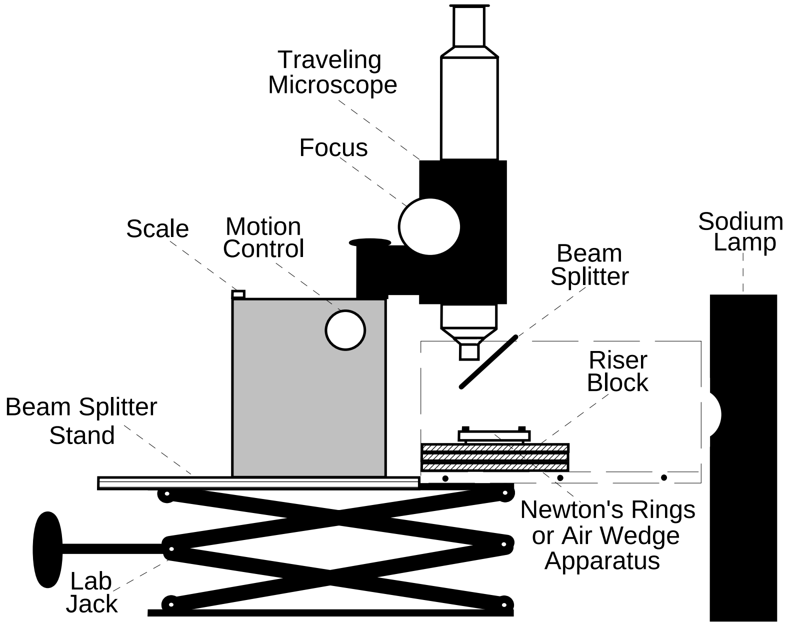

Figure 6

Experimental Procedure

1. Adjust the Newton's ring apparatus until the rings are visible and centered. This may require cleaning the lenses. To do this remove the screws which hold the two metal rings together. Remove the two lenses to be cleaned, remembering how they were positioned. If the two lenses are not put back together properly a saddle pattern, as opposed to a ring pattern will result. Before using the lens paper and lens cleaning fluid to clean off finger prints, gently blow away any dirt to avoid scratching the lenses. The ring pattern may be centered by adjusting the tension in the screws. Be careful not to tighten the screws too much or the observed pattern will not be caused by the shape of the two lenses, but by the distortion of the lenses caused by the pressing metal rings.

2. Measure the separation between the rings in order to determine the radius of curvature of the convex lens. The path difference at the point of contact between the convex and flat lenses is not necessarily zero. Even if there is no dust and the surfaces actually touch, the pressure between the lenses causes distortion. Hence it is better to measure the distance from say ring 10, as opposed to taking measurements from the center. Since the lines are so closely spaced, it is more accurate to measure the distance between a number of rings. The Newton's ring apparatus should be placed so that light from the sodium lamp, 589.3nm, is reflected down onto it by the beam splitter. Using the traveling microscope, measure the distance from a set ring, such as ring 10, to at least five other rings. These rings should be separated from each other by at least five rings. When using this method, m is the number of rings between the two measurements, and r_m, the separation.

3. Determine the thickness of a hair using the air wedge method. Make sure that the two glass slides are clean. Place a hair between the two glass slides near one end. Place the clip at the other end. Put the air wedge apparatus on the riser block so that the clip hangs off the edge. This is done so that the air wedge apparatus does not make another air wedge with the riser block. Place the riser block and air wedge apparatus under the beam splitter and use the traveling microscope to observe the interference bands produced using the sodium lamp. The bands observed should be pretty much parallel with the hair. If the bands are almost perpendicular with the hair, then the slides need to be cleaned again. Line the microscope up with the interference bands, not the hair. Starting at the edge of the hair closest to the clip, measure the distance between the clip and a determined m^{th} band. Repeat the measurement for at least five different m values. The value of m used should be no smaller then 10. Measure the distance between the hair and the edge of the clip.

4. Repeat step 3 using a piece of aluminum foil to determine its thickness.

5. With a micrometer, measure the thickness of the hair and the aluminum foil. Be sure not to over tighten the micrometer. The clutch at the end of the micrometer must be used to get a correct reading. This ensures a gentle uniform pressure for all micrometer measurements.

6. Perform any optional part of the experiment as desired.

Error Analysis

The wavelength value, λ, along with the number of bands, m, are taken to be exact. This leaves the distance measurements, which do contribute to the error in the final result. Normally when error is taken off a vernier scale it is taken as half the smallest division. Because of complications resulting from lining the microscope up with the bands, error should be considered to be at least the value of the smallest division. When using the micrometer, error should be taken to be half the smallest division.

The values obtained for the curvature of the convex lens, or for the thickness of thin objects, may not be entirely correct. This is because there are some uncontrollable factors that effect the results. Fizeau bands can be used to determine the flatness of an optical object. This means that if an object is not perfectly flat, distortions in the band pattern will result. The glass slides used for the air wedge are not optical flats, therefore, some of the band patterns observed may be due to the distortions in the glass and not in the placement of the thin object. Any dust on the slides or the lenses will also cause variation in the observed ring or band pattern. There is also no guarantee that for the Newton's rings the two lenses are actually touching, or for the air wedge that the slides are making contact with the thin objects. This will cause the values to appear larger then they actually are. A main source of unavoidable error comes from the use of clamping devices, the metal rings on the Newton's rings apparatus, and the clip on the air wedge apparatus. Both of these clamping devices are required to ensure that the two lenses or slides are actually making contact, but they also create distortions.

The following should be handed into your laboratory instructor

1. (1 mark) A derivation of Equation 19.

2. (2 marks) A table which includes the values m, r_m, r_m^2, along with their associated error, for the trials done to determine the radius of curvature of the convex lens.

3. (2 marks) A graph of r_m^2 versus m and obtain a regression equation for the straight line, including the uncertainty. From this equation recover the value for the radius of curvature of the convex lens along with its uncertainty.

4. (4 marks) Two tables, one for determining the thickness of a hair and one for finding the thickness of a piece of aluminum foil, which contain the values L, m, and y, along with their errors.

5. (4 marks) Two graphs of y versus m, one for the hair and one for the aluminum foil. Obtain the regression equations for the straight lines, including the uncertainty. From the equations recover the values, with uncertainty, for the thickness of the hair and the aluminum foil.

6. (2 marks) Values for the thickness of the hair and aluminum foil, along with the uncertainty, using the micrometer. Compare the values obtained with the micrometer with those obtained through optical means. Discuss which method is better for measuring thickness.

7. (2 marks) A one paragraph discussion on whether your results support or deny the wave nature of light.

8. (2 bonus marks) Even though we have been observing Newton's rings through monochromatic light, they can be observed through white light. When this is done the resulting pattern does not strictly result in Fizeau bands, since it is not just d which is changing, but λ as well. If all we were doing was varying λ, then the result would be an FECO band pattern, where color bands would be alternated with dark bands. The resulting colorful ring pattern does not follow a standard white light spectrum either, but instead follows an

interference color pattern. This pattern can be seen on the color interference chart provided in the laboratory. The resulting interference pattern is a combination of both Fizeau and FECO bands.

Observe the Newton's ring pattern in white light by placing the Newton's ring apparatus under one of the desk lamps. It is easier to observe the color pattern by using the larger Newton's ring apparatus. Starting from the center, write down the color sequence. Compare this sequence to the one on the chart provided in the laboratory. Does a black spot appear in the center? If not, where in the interference color pattern spectrum does the ring pattern start at? While still looking at the ring pattern under the white light adjust the tension in the tension screws. How does this affect the interference color pattern? Include in your discussion any other observations you make.

9. (2 bonus marks) By illuminating a thin plan parallel reflecting surface with diffuse monochromatic light, fringes of equal inclination can be observed. One way to observe this type of interference pattern is with a thin cover glass and a Sodium lamp. First remove the cover on the Sodium lamp, being careful not to touch the bulb. Place the cover slip on the table in front of the lamp. Move your head, or the cover slip, until you can see the reflection of the Sodium lamp in the cover slip. Place the diffuser in front of the Sodium lamp. Observe the interference pattern present on the surface of the cover slip. Rotate the cover slip and note what happens. Observe the appearance of the band pattern as you change your viewing angle. Replace the cover slip with a glass slide and make similar observations. How do the bands produced with the thicker glass slide compare to the bands produced with the cover slip? Include in your discussion any other observations you make.