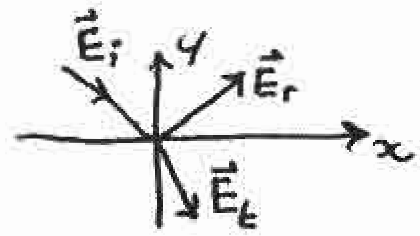
1. Consider a linear medium where $\vec{D} = \epsilon \vec{E}$



$$\vec{D}_i = \epsilon_1 \vec{E}_i = \epsilon_1 \vec{E}_{oi} e^{i(k_x x - k_y y - \omega t)}$$ for a plane wave

$$\vec{D}_r = \epsilon_1 \vec{E}_r = \epsilon_1 \vec{E}_{or} e^{i(k_x x + k_y y - \omega t)}$$

$$\vec{D}_t = \epsilon_2 \vec{E}_t = \epsilon_2 \vec{E}_{ot} e^{i(k_x x - k_y y - \omega t)}$$

at the boundary $(y=0)$: $\vec{D}_t = \vec{D}_i + \vec{D}_r$

As in the usual derivation for Snell's law the three fields must have the same dependance on r and t.

$D_{\parallel above} = D_{\parallel below}$ would afect the magnitude of $\vec{E}_{or}$ and $\vec{E}_{ot}$ but the same arguments that lead to Snell's law remain identical.

$$\vec{k}_i \cdot \vec{r} = \vec{k}_t \cdot \vec{r} \qquad k_{ix} = k_i \sin\theta_i = k_{tx} = k_t \sin\theta_t$$

$k \propto n$ so $n_i \sin\theta_i = n_t \sin\theta_t$, Snell's law is unchanged.

2. $I = \langle S \rangle = \frac{1}{2} n \epsilon_0 c E_0^2$

Flux per unit area for a flat surface $\langle S \rangle = \frac{1}{2} n \epsilon_0 c E_0^2 \cos\theta$

$$\langle S_R \rangle = \langle S_i \rangle R = \langle S_i \rangle r^2 \frac{\cos\theta_r}{\cos\theta_i} \frac{n_r}{n_i} = \langle S_i \rangle r^2$$

$$\langle S_T \rangle = \langle S_i \rangle t \frac{\cos\theta_t}{\cos\theta_i} \frac{n_t}{n_i}$$

TE case: $r^2 = \left(\frac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t} \right)^2 = \frac{n_1^2 \cos^2\theta_i + n_2^2 \cos^2\theta_t - 2 n_1 n_2 \cos\theta_i \cos\theta_t}{(n_1 \cos\theta_i + n_2 \cos\theta_t)^2}$

$$t^2 = \frac{4 n_1^2 \cos^2\theta_i}{(n_1 \cos\theta_i + n_2 \cos\theta_t)^2}$$

$$\langle S_{R_{TE}} \rangle + \langle S_{T_{TE}} \rangle = \langle S_{i_{TE}} \rangle \left[\frac{n_1^2 \cos^2\theta_i + n_2^2 \cos^2\theta_t - 2 n_1 n_2 \cos\theta_i \cos\theta_t + 4 n_1 n_2 \cos\theta_i \cos\theta_t}{(n_1 \cos\theta_i + n_2 \cos\theta_t)^2} \right]$$

$$= \langle S_{i_{TE}} \rangle \left[\frac{(n_1 \cos\theta_i + n_2 \cos\theta_t)^2}{(n_1 \cos\theta_i + n_2 \cos\theta_t)^2} \right]$$

$$= \langle S_{i_{TE}} \rangle$$

TM case: $r^2 = \frac{n_2^2 \cos^2\theta_i + n_1^2 \cos^2\theta_t - 2 n_1 n_2 \cos\theta_i \cos\theta_t}{(n_1 \cos\theta_t + n_2 \cos\theta_i)^2}$ $\quad t^2 = \frac{4 n_1^2 \cos^2\theta_i}{(n_1 \cos\theta_t + n_2 \cos\theta_i)^2}$

$$\langle S_{R_{TM}} \rangle + \langle S_{T_{TM}} \rangle = \langle S_{i_{TM}} \rangle \left[\frac{(n_2 \cos\theta_i + n_1 \cos\theta_t)^2}{(n_1 \cos\theta_t + n_2 \cos\theta_i)^2} \right]$$

$$= \langle S_{i_{TM}} \rangle$$

In both cases, energy is conserved.

3. TE case: $r_{TE} = \dfrac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t}$

I_{TE} : $\quad R_{TE} = r_{TE}^2 = \dfrac{(n_1\cos\theta_i - n_2\cos\theta_t)^2}{(n_1\cos\theta_i + n_2\cos\theta_t)^2} = \dfrac{(n_1^2\cos^2\theta_i + n_2^2\cos^2\theta_t - 2n_1n_2\cos\theta_i\cos\theta_t)}{(n_1^2\cos^2\theta_i + n_2^2\cos^2\theta_t + 2n_1n_2\cos\theta_i\cos\theta_t)}$

Similarily $\quad R_{TM} = \dfrac{(n_2\cos\theta_i - n_1\cos\theta_t)^2}{(n_1\cos\theta_t + n_2\cos\theta_i)^2} = \dfrac{(n_2^2\cos^2\theta_i + n_1^2\cos^2\theta_t - 2n_1n_2\cos\theta_i\cos\theta_t)}{(n_1^2\cos^2\theta_t + n_2^2\cos^2\theta_i + 2n_1n_2\cos\theta_i\cos\theta_t)}$

☆ $R_{Tot\,45°} = \dfrac{R_{TE} + R_{TM}}{2} = \dfrac{1}{2}\,\dfrac{((n_1^2+n_2^2)(\cos^2\theta_i + \cos^2\theta_t) - 4n_1n_2\cos\theta_i\cos\theta_t)}{((n_1^2+n_2^2)(\cos^2\theta_i + \cos^2\theta_t) + 4n_1n_2\cos\theta_i\cos\theta_t)}$

The angle of polarization with respect to the TE polarization is:

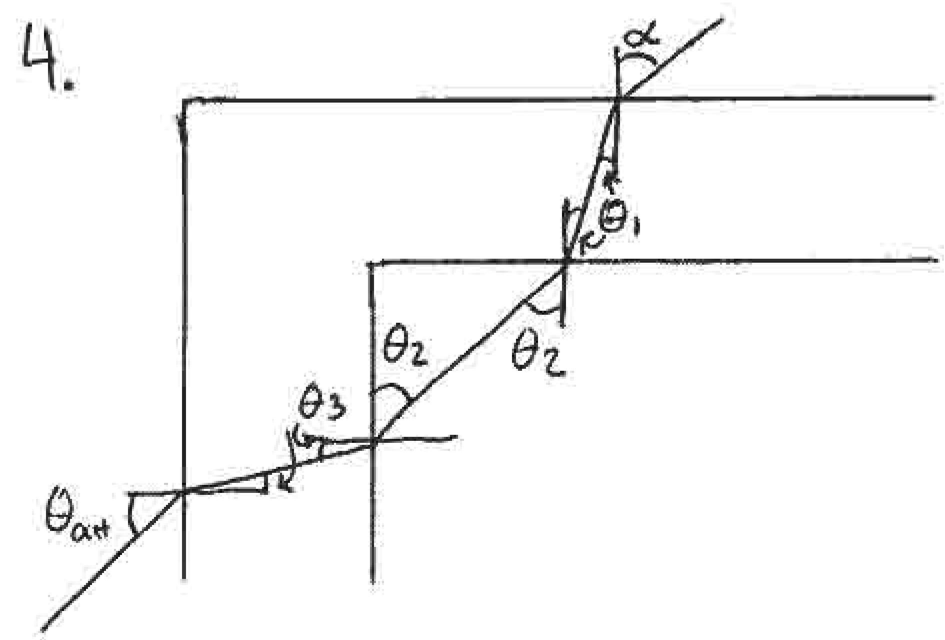
$$\tan(\theta) = \frac{r_{TM}}{r_{TE}} = \frac{(n_2\cos\theta_i - n_1\cos\theta_t)(n_1\cos\theta_i + n_2\cos\theta_t)}{(n_1\cos\theta_t + n_2\cos\theta_i)(n_1\cos\theta_i + n_2\cos\theta_t)}$$

$$\tan(\theta) = \frac{n_1n_2(\cos^2\theta_i - \cos^2\theta_t) + (n_2^2 - n_1^2)\cos\theta_i\cos\theta_t}{n_1n_2(\cos^2\theta_i + \cos^2\theta_t) + (n_1^2 + n_2^2)\cos\theta_i\cos\theta_t}$$

$$\theta_t = \sin^{-1}\left(\frac{n_1}{n_2}\sin\theta_i\right)$$

☆ We add a factor of $\frac{1}{2}$ because $I_{TE} = I_{TM} = \frac{1}{2} I_o$

4.



$n_i \sin\theta_i = n_t \sin\theta_t$

$\sin\theta_1 = \frac{1}{n_g}\sin\alpha$

$\sin\theta_2 = \frac{n_g}{n_w}\sin\theta_1 = \frac{1}{n_w}\sin\alpha$

$\sin\theta_3 = \frac{n_w}{n_g}\sin(\pi/2-\theta_2) = \frac{n_w}{n_g}\cos(\sin^{-1}(\frac{1}{n_w}\sin\alpha))$

$\sin\theta_{out} = n_g \frac{n_w}{n_g}\cos(\sin^{-1}(\frac{1}{n_w}\sin\alpha))$

$\sin\theta_{out} = n_w\sqrt{1-\frac{1}{n_w^2}\sin^2\alpha}$

a) $n_g = n_w = 1.33$ for internal reflection $\sin\theta_{out} = 1$

so $\left(\frac{1}{n_w}\right)^2 = 1 - \frac{1}{n_w^2}\sin^2\alpha$

$\Rightarrow \alpha = \sin^{-1}\left(n_w\sqrt{(1-(\frac{1}{n_w})^2)}\right)$

$\Rightarrow \alpha \leqslant 1.07\,\text{rad}$

The only internal reflection possible is on the glass-air interface.

b) $n_g = 1.5$ Now, we can have total internal reflection on the glass-air interface, or the glass-water interface.

Consider the glass-water interface:

$\sin\theta_2 = 1 = \frac{1}{n_w}\sin\alpha$ $\alpha = \sin^{-1}(1.5)$ wich is not defined.

Total internal reflection cannot occur on this interface.

Consider the glass-air interface: The expression for θ_{out} is unchanged, so again $\alpha \leqslant 1.07$ rad.

c) Consider the water-glass interface:

$$\sin\theta_3 = 1 = \frac{n_w}{n_g}\cos\left(\sin^{-1}\left(\frac{1}{n_w}\sin\alpha\right)\right)$$

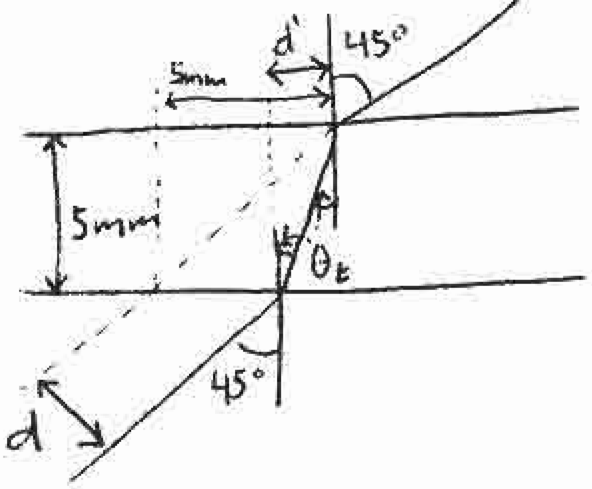
$$\alpha = \sin^{-1}\left(n_w \sin\left(\cos^{-1}\left(\frac{n_g}{n_w}\right)\right)\right) = 0.66\text{rad}$$

So we have total internal reflection for $\alpha \leqslant 0.66$rad

But, we still have $\alpha \leqslant 1.07$rad on the glass-air interface.

In all three cases we obtain internal reflection if $\alpha \leqslant 1.07$rad

5.



$$\sin\theta_t = \frac{1}{n_g}\sin(45^\circ) = \frac{1}{\sqrt{2}\,n_g}$$

$$d' = (5\text{mm})\tan\left(\arcsin\left(\frac{1}{\sqrt{2}\,n_g}\right)\right)$$

$$= (5\text{mm})\left(\sqrt{2}\,n_g\sqrt{1-\frac{1}{2n_g^2}}\right)^{-1}$$

$$d = \sin(45^\circ)\left[5\text{mm} - d'\right]$$

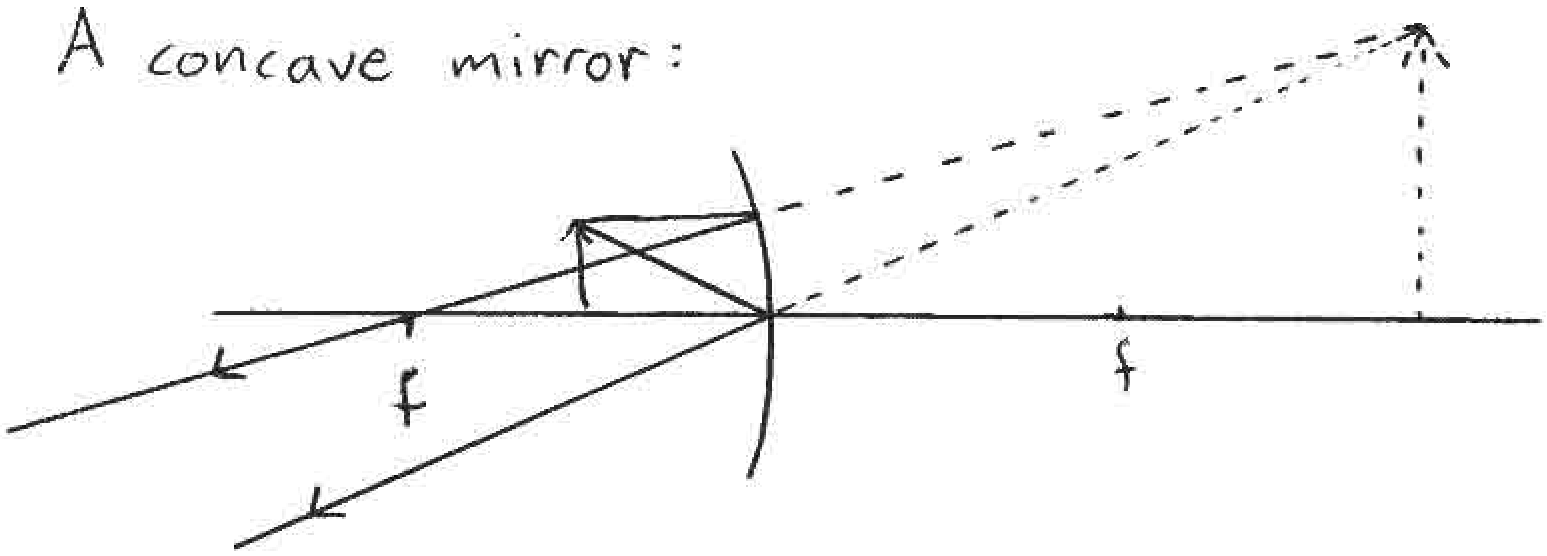
$$= \frac{5\text{mm}}{\sqrt{2}}\left(1-\left(\sqrt{2}\,n_g\sqrt{1-\frac{1}{2n_g^2}}\right)^{-1}\right)$$

$d = 1.65\text{mm}$

6. To increase the field of view that a driver observes when looking into the passenger-side rearview mirror, the mirror is given some curvature. To always produce a virtual, erect image with a larger field of view than a flat mirror, a convex mirror is used. The result is a reduced virtual image that is <u>closer</u> than the object. However, because the image <u>looks smaller</u> than it would in a flat mirror and because drivers know roughly how large vehicles are, the mind interprets the image as car-sized, but further away. Thus, a closer virtual image creates the illusion of an object that is farther.

Consider a solution consistent with geometric optics:

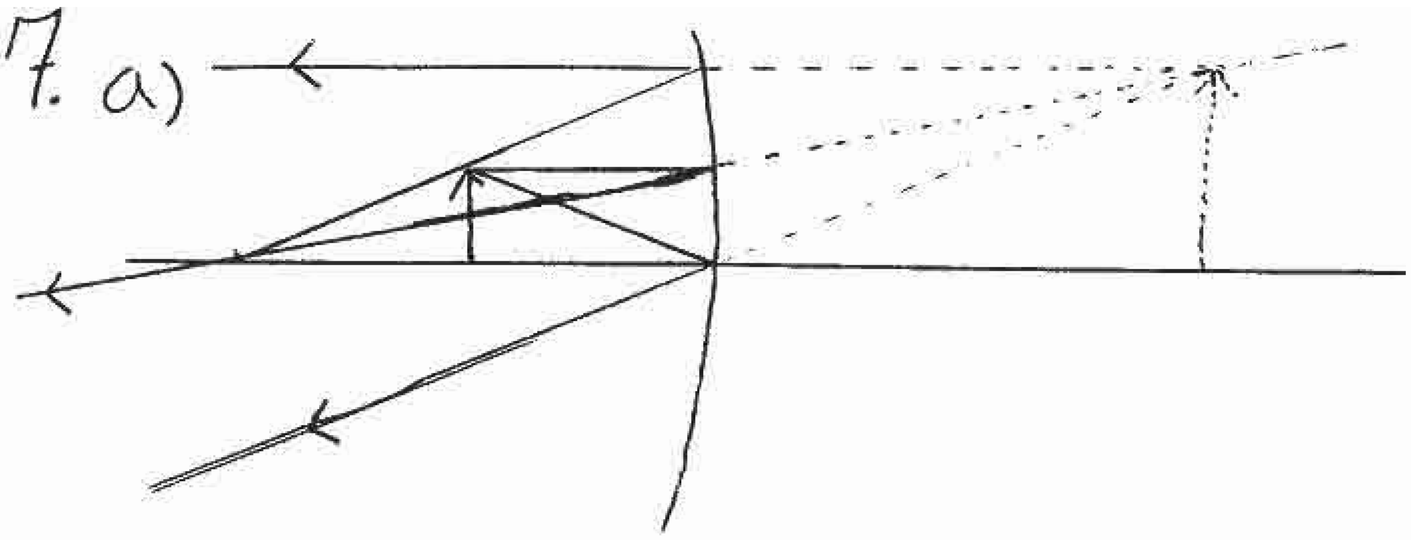
A concave mirror:



We can obtain an image that <u>is</u> virtual, erect and further than the object. In this case:

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{100\text{m}} - \frac{1}{125\text{m}} = \frac{1}{500\text{m}} \qquad f = 500\text{m}$$

7. a)



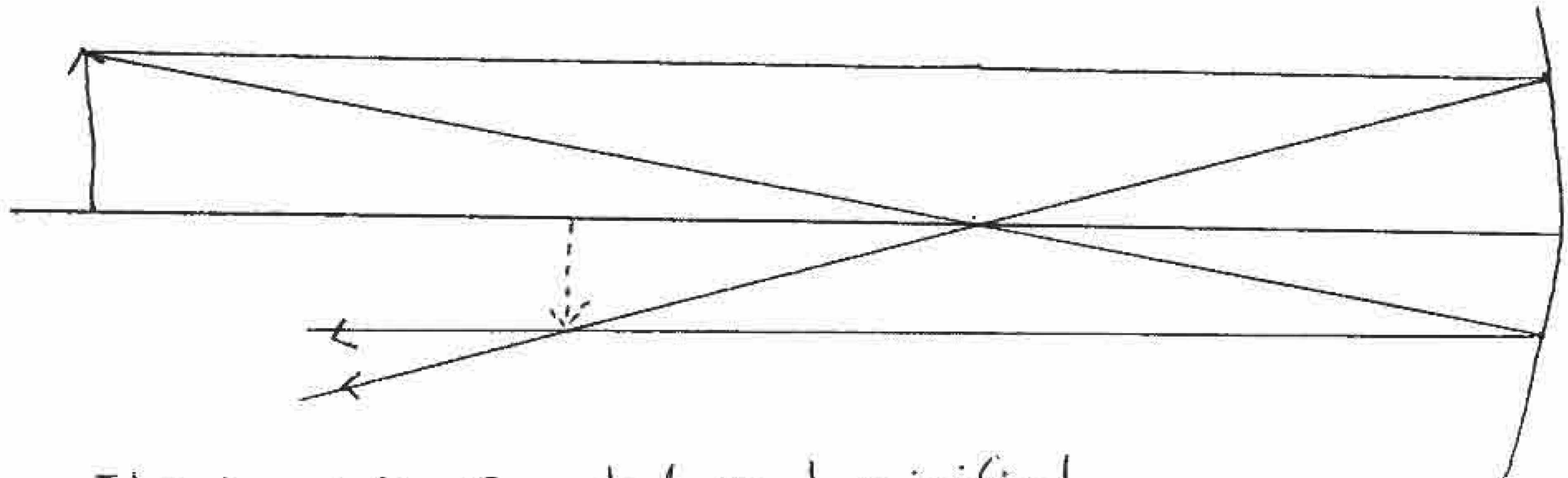
The image is
Erect, Virtual, Magnified

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} \Rightarrow \frac{1}{4cm} - \frac{1}{2cm} = -0.25\,cm^{-1} = \frac{1}{s'}$$

$s' = -4cm$

$$M = -\frac{s'}{s} = \frac{+4cm}{2cm} = 2$$

b)

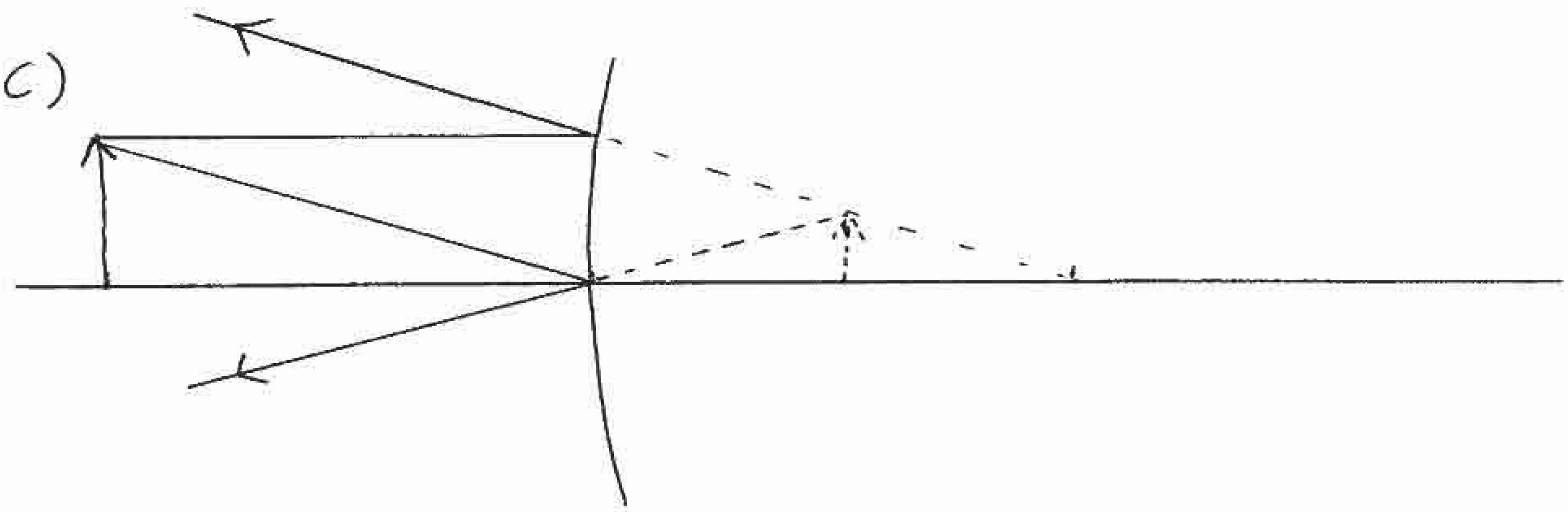


The image is: Inverted, real, minified

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} \Rightarrow \frac{1}{6cm} - \frac{1}{15cm} = 0.1cm^{-1} = \frac{1}{s'} \quad s' = 10cm$$

$$M = \frac{-s'}{s} = -\frac{2}{3}$$

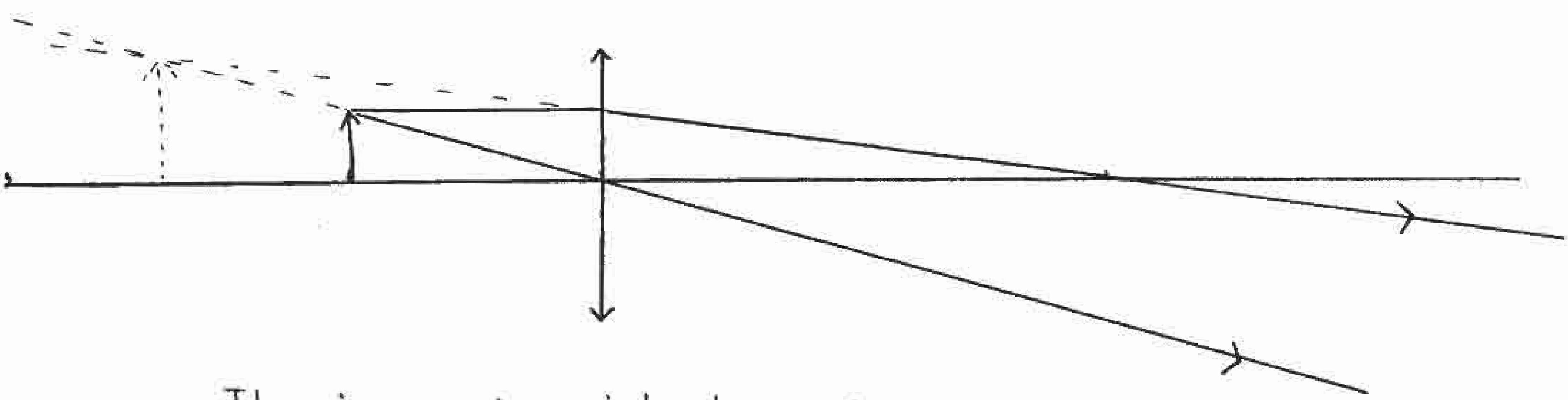
c)



The image is: Erect, Virtual, Minified

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} \Rightarrow \frac{-1}{5cm} - \frac{1}{5cm} = -0.4cm^{-1} = \frac{1}{s'} \quad s' = -2.5cm \quad M = \frac{-s'}{s} = 0.5$$

d)

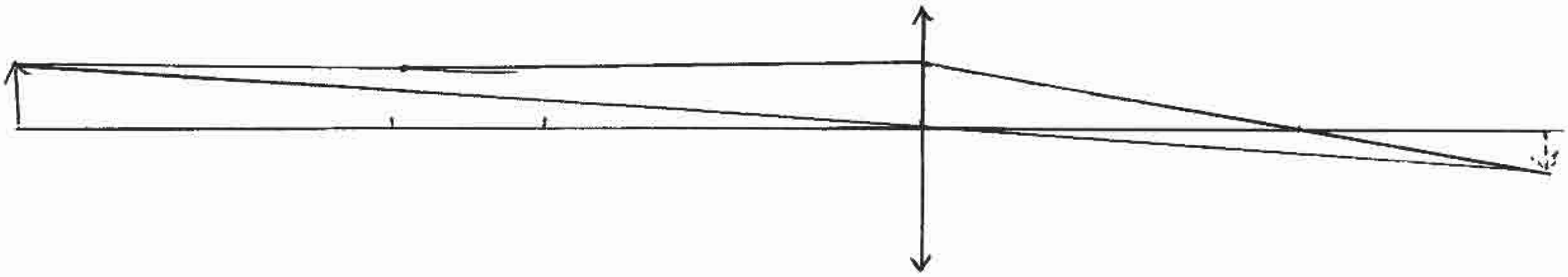


The image is virtual, erect, magnified

$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} \Rightarrow \frac{1}{6cm} - \frac{1}{3cm} = -0.17cm^{-1} = \frac{1}{s'} \quad s' = -6cm$

$M = \frac{-s'}{s} = 2$

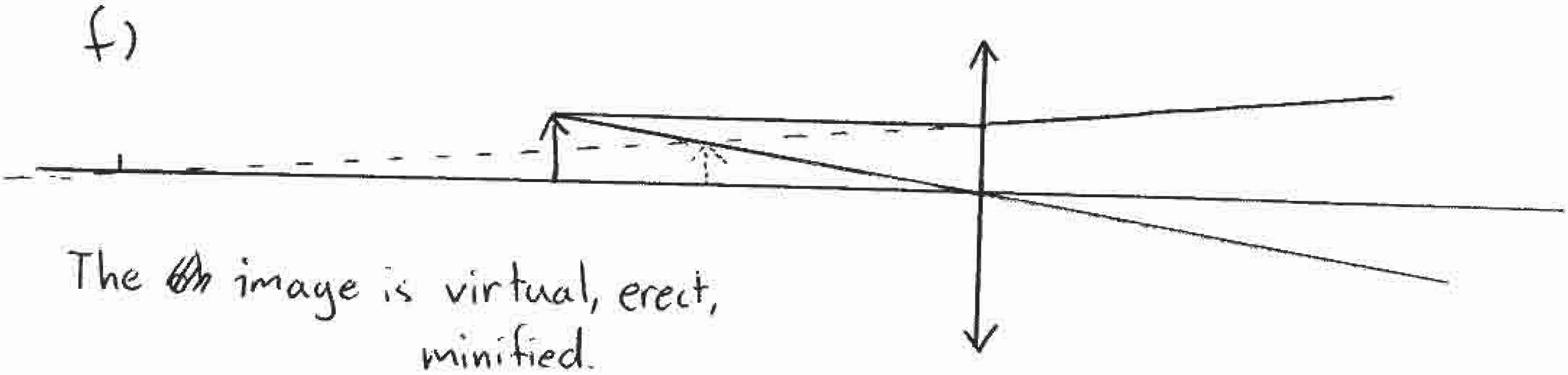
e)



The image is real, inverted, minified

$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} \quad \frac{1}{5cm} - \frac{1}{12cm} = \overset{0.117cm^{-1}}{\cancel{0.05cm}} = \frac{1}{s'} \quad s' = 8.5cm \quad M = \frac{-8.5}{12} = -0.714$

f)



The ~~th~~ image is virtual, erect, minified.

$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} \Rightarrow \frac{-1}{10cm} - \frac{1}{5cm} = -0.3cm^{-1} = s' \quad s' = -3.33cm$

$M = \frac{3.33}{5} = 0.67$

8. e) Convex spherical lens with $f = 5cm$.

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

For simplicity, we'll pick $R_1 = R_2 = R$ and we'll use a glass with $n = 1.5$

$$\frac{1}{5cm} = (0.5)\left(\frac{2}{R}\right)$$

$\Rightarrow R = 5cm$ so the lens is symmetric with curvatures of 5cm on both sides.

f) Concave spherical lens with $f = 10cm$

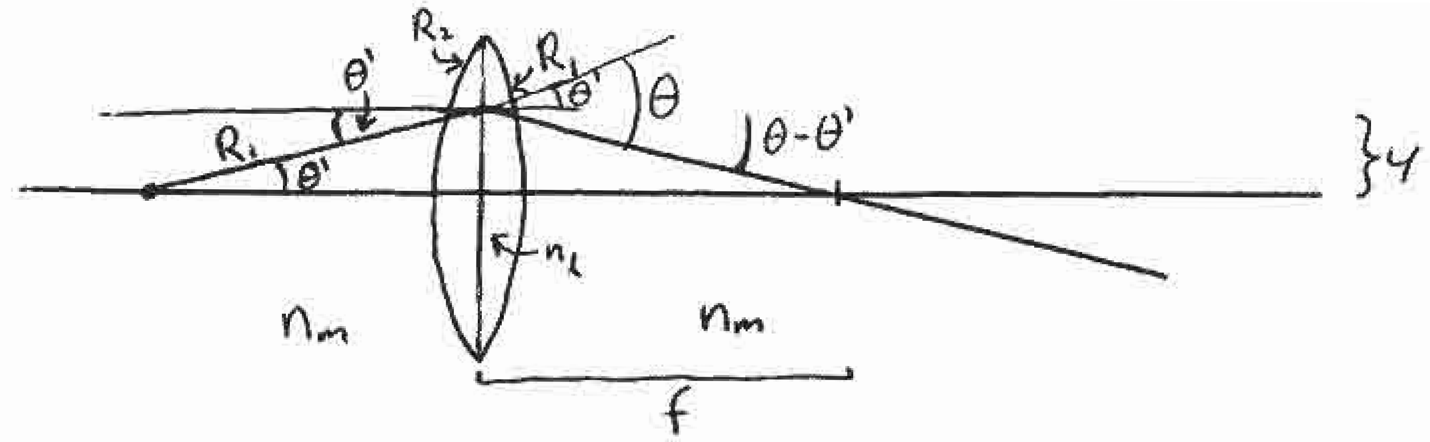
$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

Again, $R_1 = R_2 = R$, $n = 1.5$

$$\frac{-1}{10cm} = (0.5)\left(\frac{2}{R}\right)$$

Both sides will be concave with $R = -10cm$

9.



$y/R_1 = \sin\theta' \simeq \theta'$ for $\theta \ll 1$

$$n_l \sin\theta' = n_m \sin\theta \rightarrow n_l\theta' = n_m\theta$$

$$\theta - \theta' = \left(\frac{n_l}{n_m} - 1\right)\theta' = \left(\frac{n_l}{n_m} - 1\right) y/R_1$$

$$f = \frac{y}{\tan(\theta-\theta')} \simeq \frac{y}{\theta-\theta'} = \frac{R_1}{(n_l/n_m - 1)} = \frac{n_m R_1}{(n_l - n_m)}$$

$$\frac{1}{f} = \frac{n_l - n_m}{n_m}\frac{1}{R_1}$$

We know that the other lens (the other half) will add it's optical power linearly.

So $$\frac{1}{f} = \frac{n_l - n_m}{n_m}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$