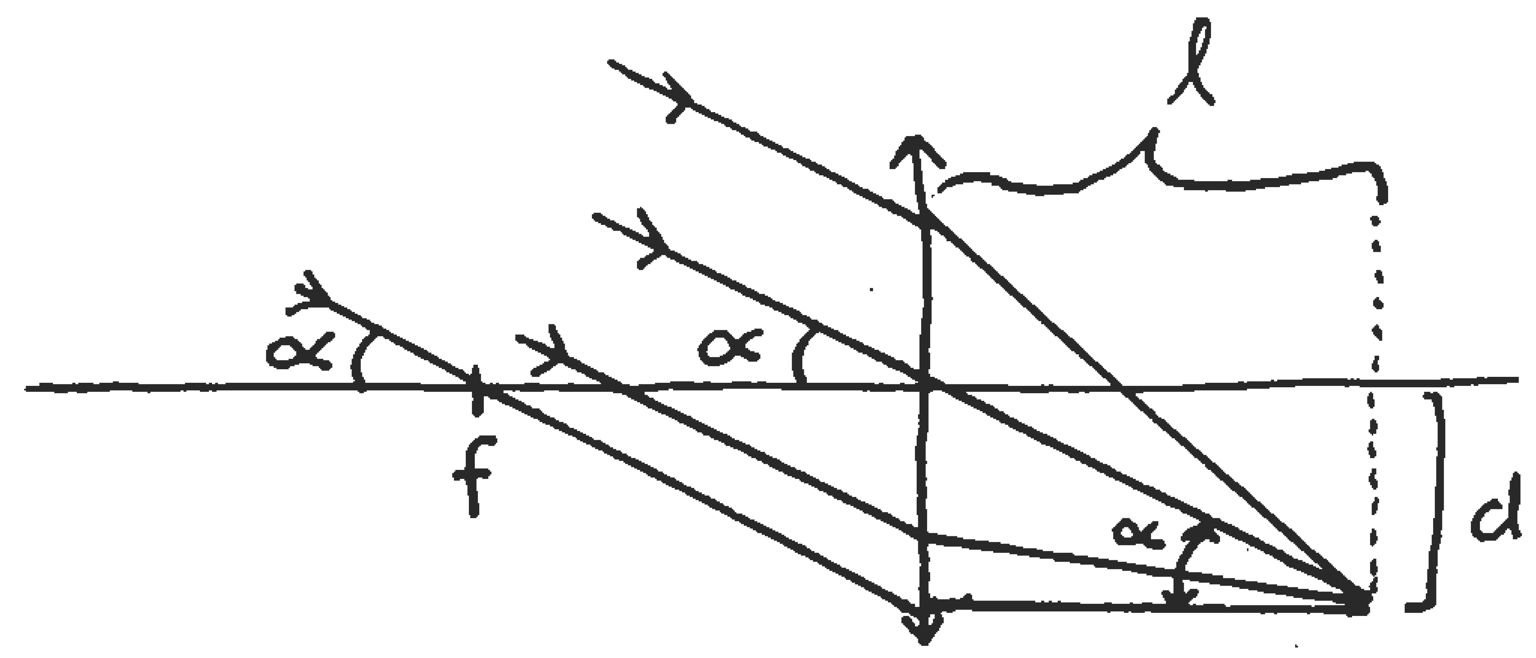
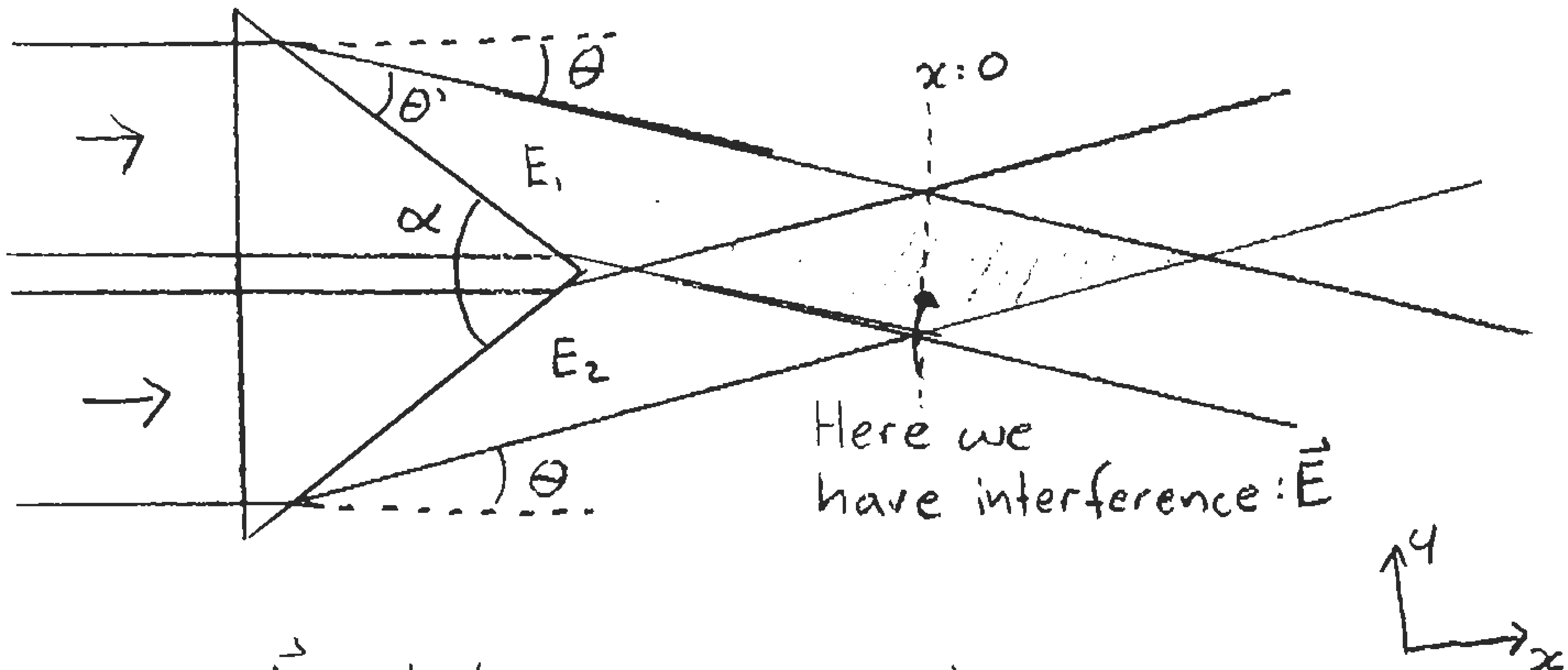
1.



Consider a ray passing through the point f (a focal length away from the lens on the optical axis) that makes an angle α with the optical axis. The lens will render it parallel with the optical axis. It will then be a distance d from the optical axis with $d = f\tan(\alpha)$. A ray passing through the center of the lens will not be deflected and will travel a distance ℓ in the direction of the optical axis before intersecting the first ray that we considered. $\ell = d(\tan(\alpha))^{-1} = f$. So, incoming parallel rays will always converge in a plane (the focal plane), a distance f away from the lens, and at a distance $d = f\tan(\alpha)$ from the optical axis.

☆ In the paraxial approximation $\tan(\alpha) \approx \alpha$.

2.



$\vec{E}_1 = \vec{E}_0 e^{i(\omega_1 t - \vec{k}_1 \cdot \vec{r})}$

$\vec{E}_2 = \vec{E}_0 e^{i(\omega_2 t - \vec{k}_2 \cdot \vec{r} + \phi)}$

Both fields are from the same laser so $\omega_1 = \omega_2$, $|\vec{k}_1| = |\vec{k}_2| = k$ and $\phi = 0$

$\vec{k}_1 = k(\cos\theta\hat{x} - \sin\theta\hat{y})$

$\vec{k}_2 = k(\cos\theta\hat{x} + \sin\theta\hat{y})$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \vec{E}_0\left[e^{i(\omega t - k(x\cos\theta - y\sin\theta))} + e^{i(\omega t - k(x\cos\theta + y\sin\theta))}\right]$$

$$= \vec{E}_0 e^{i\omega t - kx\cos\theta}\left[e^{iky\sin\theta} + e^{-iky\sin\theta}\right] \qquad \Rightarrow \text{choose } x=0$$

$$= \vec{E}_0 e^{i\omega t}\left[2\cos(ky\sin\theta)\right]$$

$$\langle \vec{E} \rangle_T = 2\vec{E}_0 \cos(ky\sin\theta)$$

$$\langle I \rangle_T = \langle E_T \rangle^2 = 4|\vec{E}_0|^2\cos^2(ky\sin\theta)$$

The period of $\cos^2$ is π so $\Delta y = \frac{\pi}{k\sin\theta}$ is the fringe period

$\lambda = \frac{2\pi}{k}$ so $\Delta y = \frac{\lambda}{2\sin\theta}$

$\sin\theta' = n\sin(\tfrac{1}{2}\alpha)$ $\qquad$ $\theta' = \frac{\pi}{2} - \theta - \left(\frac{\pi}{2} - \frac{\alpha}{2}\right)$

$\qquad\qquad\qquad\qquad\qquad = \frac{\alpha}{2} - \theta$

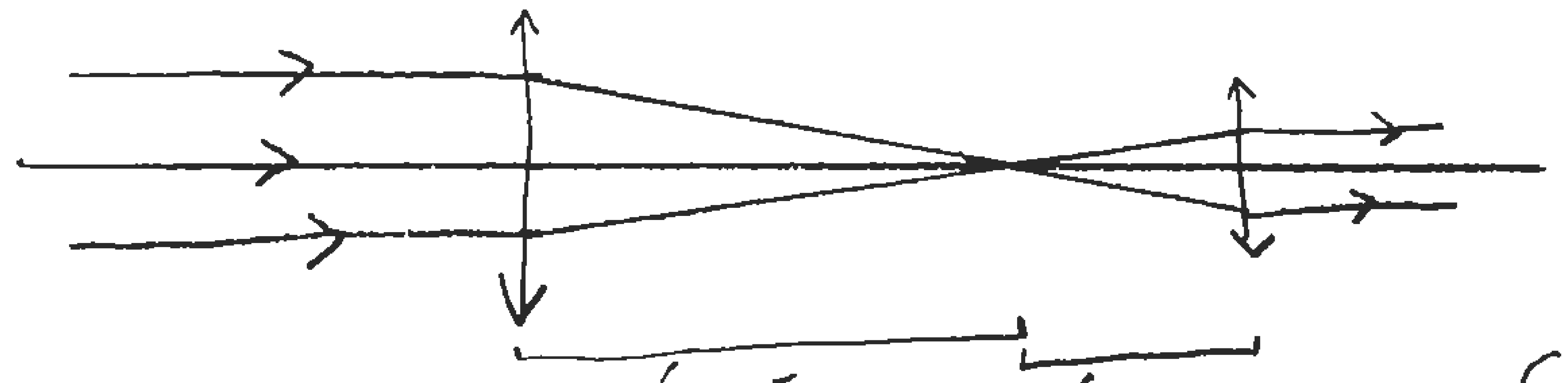
$\Rightarrow \sin\left(\frac{\alpha}{2} - \theta\right) = n\sin\left(\frac{\alpha}{2}\right)$

In the paraxial approximation: $\frac{\alpha}{2} - \theta \approx n\frac{\alpha}{2} \Rightarrow \theta \approx \frac{\alpha}{2}(1-n)$

The fringe period is $\Delta y \approx \frac{\lambda}{\alpha(1-n)}$

3. Water in front of the eye reduces the strength of the eye's lens (the difference in indicies of refraction is less than if the eye was in air). This implies that the person's eye has too strong a lens, a condition known as nearsightedness.

4. a)



$f_1 = 500mm$ $f_2 = 10mm$ $f_1 + f_2 = \boxed{510mm}$

b) $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$

Image formed by lens 1: $\frac{1}{s_1'} = \frac{1}{f_1} - \frac{1}{s_1} = \frac{1}{0.5m} - \frac{1}{100m} = 1.99m^{-1}$

$s_1' = \frac{100}{199} m$

We want lens 2 to create a virtual image 2m from lens 2 (the eyepiece).

$\frac{1}{s_2} = \frac{1}{f_2} - \frac{1}{s_2'} = \frac{1}{0.01m} - \frac{1}{(-2m)} = 100.5m^{-1} \Rightarrow s_2 = \frac{2}{201} m$

We need an object $\frac{2}{201}m$ from lens 2, this object is the real image formed by lens 1.

$d = s_1' + s_2 = \left(\frac{100}{199} + \frac{2}{201}\right)m = \boxed{512mm}$

c) Glasses of -2 diopters cause an object at infinity to appear as if it was 50cm in front of the eye (or would to someone who didn't need glasses!) The telescope should form a virtual image of the star at 50cm.

$\frac{1}{s_1'} = \frac{1}{f_1} - \frac{1}{s_1} = \frac{1}{0.5m} - 0 = \frac{1}{0.5m}$ $s_1' = 0.5m$

$\frac{1}{s_2} = \frac{1}{f_2} - \frac{1}{s_2'} = \frac{1}{0.01m} - \frac{1}{(-0.5m)} = 102m^{-1}$ $s_2 = 0.0098m$ $\boxed{d = s_1' + s_2 = 509.8mm}$

The person sees the same image as someone who didn't wear glasses would see through a 510mm telescope. $\boxed{M = -f_o/f_e = -50}$

5. a) $T = t^2 \frac{n_2}{n_1} = \left(\frac{2n_1}{(n_1+n_2)}\right)^2 \frac{n_2}{n_1}$

$$= \frac{4n_1n_2}{(n_1+n_2)^2}$$

$$T_{total} = T_{air\text{-}water} \cdot T_{water\text{-}glass} = \frac{4n_w}{(1+n_w)^2}\frac{4n_wn_g}{(n_w+n_g)^2}$$

$$T_{total} = \left(\frac{4n_w\sqrt{n_g}}{(1+n_w)(n_w+n_g)}\right)^2 = \boxed{0.976} \text{ for } n_w = 1.33,\ n_g = 1.5$$

b) $T = 1 - R$ we can find R after destructive interference.

$$r = \frac{n_2 - n_1}{n_1+n_2}$$

$$r_{air\text{-}water} = \frac{1-n_w}{1+n_w} \qquad r_{water\text{-}glass} = \frac{n_w - n_g}{n_w+n_g}$$

$$E_r = E_0 r_{air\text{-}water} - E_0 t_{air\text{-}water} r_{water\text{-}glass}$$

$$= E_0 (r_{aw} - t_{aw} r_{wg})$$

$$= E_0 \left(\frac{n_w - 1}{1+n_w} - \frac{2}{1+n_w}\frac{n_g - n_w}{n_w+n_g}\right)$$

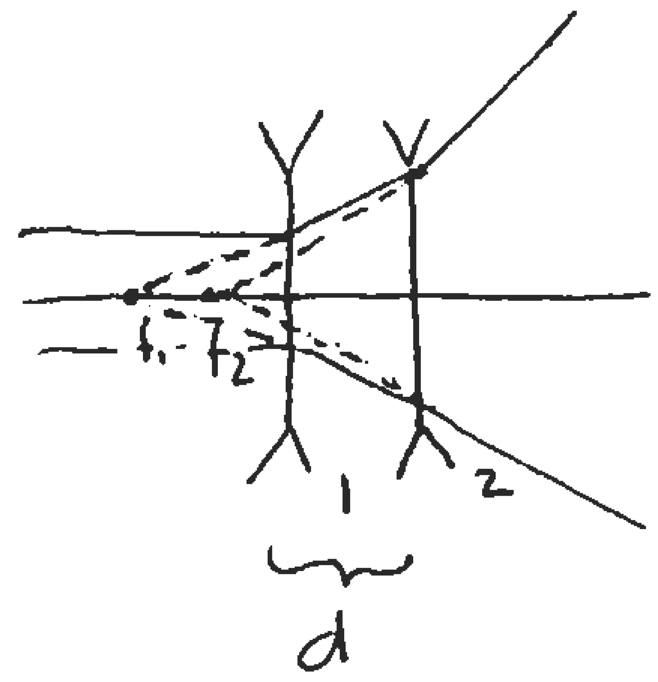
$$= E_0 r_{eff}$$

$$R = r_{eff}^2 = 0.008$$

so $\boxed{T = 0.992}$

Condition for destructive interference:
We get a π phase shift on reflection thus the path length must be an integer number of wavelengths: $d = \frac{1}{2}\frac{\lambda}{n_w}m = \frac{\lambda}{2.66}m$

☆Note: This is how anti-reflection coatings work!

6. 

Consider the effect of lens 1:

$$\frac{1}{i} = \frac{1}{f} - \frac{1}{o} \qquad o = \infty \Rightarrow i = f_1$$

Now, the effect of lens 2: $o' = -f_1 + d$

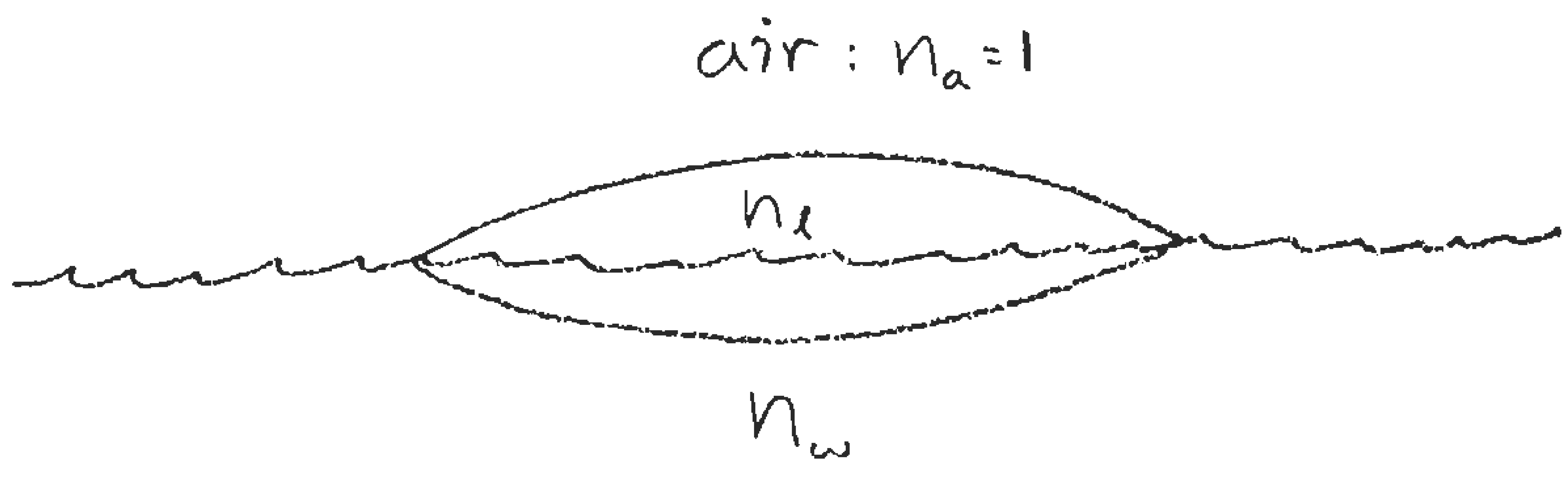
So $$\frac{1}{f_1 - d} = \frac{1}{} \qquad \frac{1}{i'} = \frac{1}{f_2} - \frac{1}{-f_1 + d}$$

as $d \to 0$ $$\frac{1}{i'} = \frac{1}{f_2} + \frac{1}{f_1}$$

so the effective optical power $\frac{1}{f}$ is $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$

7. The system can be considered to be composed of two close lenses with optical powers that add.

From assignment # 2: $\dfrac{1}{f} = \dfrac{n_\ell - n_m}{n_m} \dfrac{1}{R}$



$$\frac{1}{f_a} = \frac{n_\ell - 1}{R_1} \qquad \frac{1}{f_w} = \frac{n_\ell - n_w}{n_w} \frac{1}{R_2}$$

$$\frac{1}{f} = \frac{1}{f_a} + \frac{1}{f_w} = \frac{n_\ell - 1}{R_1} + \frac{n_\ell - n_w}{n_w} \frac{1}{R_2}$$

$$f = \left[(n_\ell - 1) \frac{1}{R_1} + \left(\frac{n_\ell}{n_w} - 1 \right) \frac{1}{R_2} \right]^{-1}$$

From reversibility the focal length is the same when observed from either side.

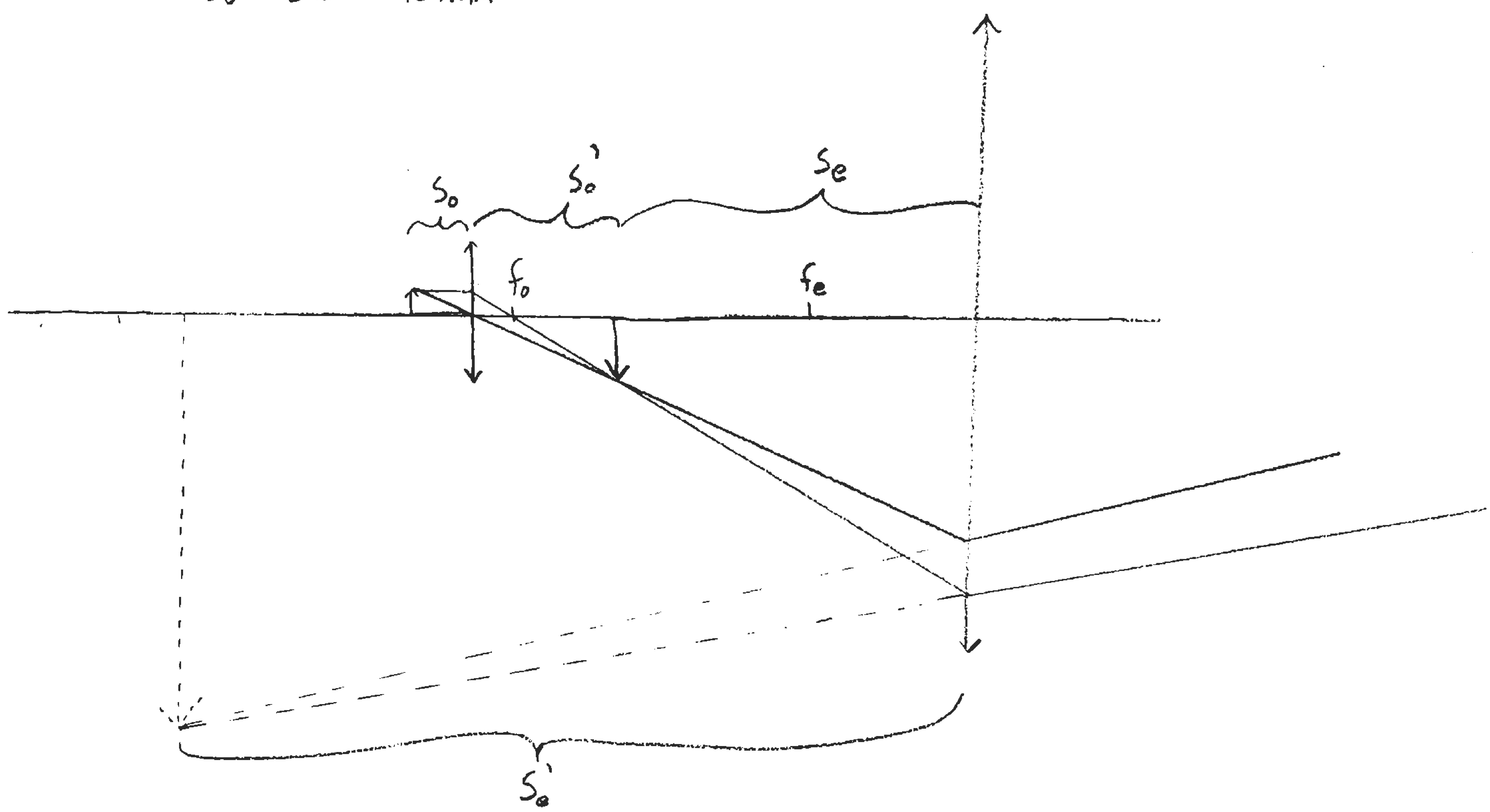
8. $\frac{1}{f_o} = \frac{1}{S_o} + \frac{1}{S_o'} \Rightarrow \frac{1}{S_o} = \frac{1}{5mm} - \frac{1}{S_o'}$

$\frac{1}{f_e} = \frac{1}{S_e} + \frac{1}{S_e'}$, $\quad S_e = 150mm - S_o'$

$\frac{1}{S_e'} = \frac{1}{25mm} - \frac{1}{150mm - S_o'}$ $\quad S_e' = -250mm \rightarrow$ We require a virtual image

$\frac{1}{25mm} + \frac{1}{250mm} = \frac{1}{150mm - S_o'} \Rightarrow S_o' = 127mm$

$\frac{1}{S_o} = \frac{1}{5mm} - \frac{1}{127mm} \Rightarrow S_o = 5.2mm$



$M = M_o M_e = \left(-\frac{S_o'}{S_o}\right)\left(-\frac{S_e'}{S_e}\right) = 265$

$M \approx \frac{L S_e'}{f_o f_e} = 300$

9. We want to increase $M \approx \frac{L S_e'}{f_o f_{e_{eff}}}$

$\frac{1}{f_{e_{eff}}} = \frac{1}{f_e} + P$ where P is the optical power of the glasses

We want to increase $\frac{1}{f_{e_{eff}}}$ so we use +5diopter glasses.

$M \approx \frac{L S_e'}{f_o}\left(\frac{1}{f_e} + 5\right) = 338$