University of Calgary
Winter semester 2008

PHYS 471: Optics

Homework assignment 6

Due Wednesday, April 16, 2008

Problem 6.1. A Google Earth satellite photo of a certain area has a resolution of $a = 30$ cm. Estimate the diameter of the objective lens of the camera assuming that the satellite is at an altitude of $H = 300$ km.

Problem 6.2. A right circularly polarized laser beam propagates through a flat glass plate ($n = 1.5$). Find the Jones vector of the transmitted light (use the most convenient reference frame) if the angle of incidence is $\theta_i = 45°$.

Problem 6.3. By applying the rotation matrix to the states' Jones vectors, show that the right and left circular polarization states are invariant with respect to the choice of the coordinate axes.

Problem 6.4. Consider a laser beam elliptically polarized clockwise (as viewed from $z = -\infty$, propagation being in the positive z direction) with the polarization ellipse's major axis at angle $\theta = 60°$ to the x-axis. Suppose a polarizer is placed in the beam path. When the polarizer is rotated, the transmitted intensity is found to vary between I_1=20 W/m^2 and I_2=5 W/m^2.

a) What are the polarizer's optical axis's orientations at which the transmitted intensity is minimized and maximized?

b) Write the normalized Jones vector of the initial wave. Draw the polarization ellipse.

c) Write the polarizer's Jones matrix as a function of the angle φ between the polarizer's optical axis and the x-axis.

d) Write the expression and plot the transmitted intensity as a function of φ. What is the transmitted intensity at $\varphi = 45°$?

e) The polarizer is replaced by a half-wave plate with its optical axis along x. What is the Jones vector of the transmitted wave? Draw its polarization ellipse. How will the handedness of polarization be affected?

Problem 6.5. The wave defined in the previous problem is subjected to the action of a half-wave plate at angle $\theta/2$ and a quarter-wave plate at angle 0.

a) Find the Jones matrices of both wave plates.

b) Find the Jones vectors between the waveplates and after the second waveplate.

c) Show that the final wave's polarization is linear. Give a verbal argument why it is so.

d) Find the final wave's intensity and polarization angle.

e) Find the angle of an additional half-wave plate that would make the wave vertically polarized.

(**Hint:** your answers may depend on the relation between the quarter-wave plate's ordinary and extraordinary refraction indices).

Problem 6.6. The extraordinary index of refraction of a uniaxial birefringent crystal is the following function of the angle θ between the propagation direction and the crystal's optic axis:

$$n_e(\theta) = n_o\sqrt{\frac{1+\tan^2\theta}{1+(n_o/n_{e0})^2\tan^2\theta}},$$

where n_o is the ordinary index and n_{e0} the extraordinary index at $\theta = \pi/2$. Consider second-harmonic generation $\lambda_1 = 1\ \mu\text{m} \rightarrow \lambda_2 = 0.5\ \mu\text{m}$ in beta-barium borate (BBO), one of the most popular nonlinear crystals. Find the index matching angle θ_0 given that $n_o(\lambda_1) = 1.656,\ n_{e0}(\lambda_1) = 1.543,\ n_o(\lambda_2) = 1.678,\ n_{e0}(\lambda_2) = 1.557$. Which of the two waves should be ordinary, and which extraordinary in order to achieve phase matching?

Problem 6.7. Consider a gas of free electrons driven by an electromagnetic field [Eq. (3.65) in the textbook]. Because the electrons are moving, they will experience the Lorentz force due to interaction with the magnetic component of the field. Find the magnitude and direction of this force. Use this finding to calculate the average light pressure on the gas (assume a certain density and sample length, such that the overall absorption is small). Make an alternative calculation of the light pressure as follows: from the complex susceptibility, determine the absorption coefficient; the absorbed photons transfer their momentum to the gas. Verify that the two calculations lead to the same result.