University of Calgary
Winter semester 2008

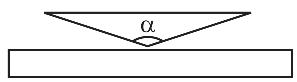
PHYS 471: Optics

Homework assignment 4

Due Wednesday, March 12, 2008

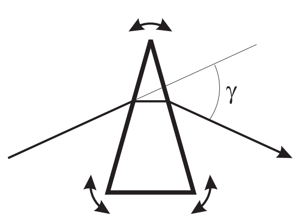
Problem 4.1. A 5-cm fish is in the center of a spherical aquarium. What size will it appear to an outside viewer? The aquarium walls are thin, the index of water is $n = 4/3$. **Hint:** Consider a layer of water in front of the fish as a combination of a thin spherical lens and a thick rectangular block.

Problem 4.2. An isosceles glass triangular prism with an apex angle $\alpha = 179.9°$ is pressed against a flat glass plate as shown in the figure. Find the shape of the Fizeau bands when reflected light is observed from the top. Find the position of the mth bright band.



Problem 4.3. A laser beam with $\lambda = 632$ nm propagates inside glass ($n = 1.45$) towards a flat interface with air at incidence angle $\theta_i = 45°$ and experiences total internal reflection at that interface. Find the expression for the evanescent wave. At which distance from the glass surface will its intensity decrease by a factor of 2? Assume that the interface is in the xy plane, the plane of incidence is xz.

Problem 4.4. A laser beam propagates through the apex corner of an isosceles triangular prism (Fig.). The position of the laser is constant, but the prism can be rotated around its axis. Show that the deviation γ of the outgoing beam from its original direction is minimized when the prism is situated so that beam inside the prism is parallel to the base of the triangle.



Problem 4.5. Find and plot Fourier transforms of functions

a) $f(x) = \delta(x)$;

b) $f(x) = \delta(x - a/2) + \delta(x + a/2)$;

c) $f(x) = \begin{cases} 1 \text{ for } |x|<b/2 \\ 0 \text{ for } |x|\geq b/2 \end{cases}$;

d) $f(x) = \begin{cases} 1 \text{ for } -a/2-b/2<x<-a/2+b/2 \text{ and } a/2-b/2<x<a/2+b/2 \\ 0 \text{ otherwise} \end{cases}$,
with $a \gg b > 0$. **Hint:** use the previuous two results and the rule for the Fourier transform of a convolution;

e) $f(x) = e^{-(x/b)^2}$;

Problem 4.6. A Fabry-Perot cavity is formed by two mirrors of reflectivity r^2 (such that $1 - r^2 \ll 1$) situated at distance l from each other. Inside the cavity, there is an attenuator with intensity absorption $L \ll 1$. Find the FWHM linewidth of the cavity in terms of the optical frequency ω as well as the minimum and maximum cavity transmission coefficients. **Hint:** an absorber can be modeled as a beam splitter with transmission L and reflectivity $1 - L$.

Problem 4.7. Light from a white source is filtered with a monochromator and sent into a Michaelson interferometer. The transmission of the monochromator $T(\lambda)$ is a top-hat function[1] of the wavelength with a width of $\delta\lambda = 1$ Å centered at $\lambda = 795$ nm.

a) Find the transmission function $T(\omega)$ of the monochromator in terms of the optical frequency. Find its width $\delta\omega$. **Hint:** because $\delta\lambda$ is small, you can assume a linear relation between ω and λ in the region where $T(\lambda)$ is substantially nonzero.

b) Find the interference pattern $I(x)$ at the output of the interferometer as a function of the path length difference x. Find the visibility as a function of x.

c) Estimate the coherence time τ_c the monochromator output based on the value of x at which the visibility drops down to zero for the first time. Verify the relation $\tau_c \times \delta\omega \sim 1$.

[1] The *top-hat function* is equal to zero everywhere except a certain interval, where it is equal to one.