University of Calgary
Winter semester 2008

PHYS 471: Optics

Homework assignment 5

Due Wednesday, March 26, 2008

Problem 5.1. A Fabry-Perot cavity is formed by two mirrors of reflectivity $r^2 = 99\%$ placed at distance $l = 15$ cm from each other. Find the FSR, FWHM (full width at half-maximum) linewidth, and the finesse of the cavity in terms of the optical frequency ν. Will this cavity be able to distinguish two $\lambda = 790$ nm laser lines situated $\delta\lambda = 0.0001$ nm apart?

Problem 5.2. A Fabry-Perot cavity is formed by two mirrors of reflectivity r^2 (such that $1 - r^2 \ll 1$) placed at distance l from each other. Inside the cavity, there is an attenuator with intensity absorption $L \ll 1$. Find the FWHM linewidth of the cavity in terms of the optical frequency $\omega = 2\pi\nu$ as well as the minimum and maximum cavity transmission coefficients. Plot them using the data from the previous problem. **Hint:** an absorber can be modeled as a beam splitter with transmission L and reflectivity $1 - L$.

Problem 5.3. Show that the optical intensity in the focal plane of a convex lens is a scaled, squared Fourier transform of the field incident on the lens:

$$I(x', y') \propto |\tilde{E}(\alpha x', \alpha y')|^2, \tag{1}$$

where

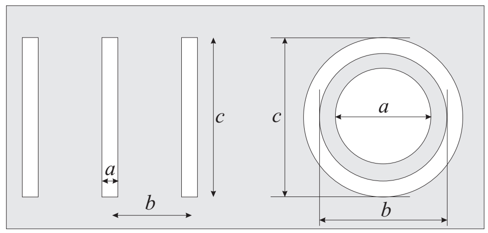
$$\tilde{E}(k_x, k_y) = \frac{1}{(2\pi)^2} \int\limits_{-\infty}^{+\infty} \int\limits_{-\infty}^{+\infty} E(x, y) e^{ik_x x + ik_y y} \, dx \, dy, \tag{2}$$

(x, y) and (x', y') denote coordinates in the plane of the lens and the focal plane of the lens, respectively. Find α if the focal length is f and the wavelength is λ. **Hint:** note that each Fourier component (k_x, k_y) of the incident field is a plane wave and will be focused into a single point in the focal plane of the lens (Problem 3.1).

Problem 5.4. Calculate, as a function of the transverse position, the Fraunhofer diffraction intensity pattern from

a) three vertical slits of width $b = 0.1$ mm with centers positioned at $(-a, 0)$, $(0, 0)$, and $(a, 0)$ with $a = 0.5$ mm. The length of each slit is $c = 1$ mm;

b) a round hole of a $a = 0.3$ mm diameter surrounded by a concentric, ring-shaped hole of $b = 0.4$ mm and $c = 0.5$ mm inner and outer diameters, respectively on a wall located $L = 10$ m away. Plot the cross-section of the pattern at $y' = 0$. The initial laser is a plane wave; the wavelength is $\lambda = 532$ nm.



Problem 5.5. What will happen to the Fraunhofer diffraction intensity distribution if the diffracting screen

a) shifts by distance d in the negative x direction;

b) its x-dimension shrinks, and its y-dimension extends by a factor of 2?

The initial laser is a plane wave.

Problem 5.6. A geometric image obtained with a camera obscura (pinhole camera) becomes clearer when the pinhole diameter is reduced. On the other hand, if the pinhole is too small, ray divergence due to diffraction will compromise the clarity and brightness. Estimate the optimal diameter of the objective pinhole in a camera obscura of a $L = 1$ m depth.

Problem 5.7. Using the considerations similar to those in the previous problem, estimate the optimal size of the input slit of a visible light spectrometer which utilizes a grating of a $D = 10$ cm width. The collimating lens has a focal length of $L = 1$ m. Show that the optimal width of the slit is such that the light incident on this slit will generate a diffraction pattern of width D. Estimate the resolution $\Delta\lambda$ and the resolving power R of this spectrometer.

Problem 5.8. In the previous problem, the grating is replaced by an equilateral prism of $D = 10$ cm side length. Estimate the minimum resolved wavelength and the resolving power of this spectrometer. To determine the dispersion, use the following data: the index of refraction of flint glass equals 1.7076 at a wavelength of 656.3 nm (red) and 1.7328 at 486.1 nm (blue).

Problem 5.9. A one-dimensional phase grating consists of alternating blocks of glass of indices n_1 and n_2, thickness a and width $d/2$. Find the ratio between the intensities in the first and zeroth diffraction orders. Neglect reflection.