Phys 471: Optics

Final examination

Solutions

1

$$I = \frac{c\,\varepsilon_0}{2} E_0^2 = \frac{\hbar\omega}{\tau(\pi w^2)}$$

$$E_0 = \sqrt{\frac{2\hbar\omega}{c\,\varepsilon_0\,\tau\,(\pi w^2)}} = \sqrt{\frac{2\hbar\, 2\pi c/\lambda}{c\,\varepsilon_0\,\tau\,\pi w^2}} = \sqrt{\frac{4\hbar}{\varepsilon_0\,\tau\,w^2\lambda}} = \sqrt{\frac{4\cdot 10^{-34}}{10^{-11}\,10^{-12}\,10^{-12}}}\ \frac{V}{m}$$

$$= 6\ V/m$$

2

$E_1 \propto \sqrt{P}$

$E_2 \propto \sqrt{100P} = 10 E_1$

a) $I_{max} \propto (E_1 + E_2)^2 = 121 E_1^2$

$I_{min} \propto (E_1 - E_2)^2 = 81 E_1^2$

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = 0.198$$

b) Let E_2 be polarized along the x-axis

$E_{1x} = E_1/2$

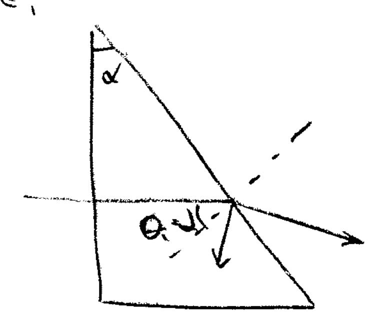
$E_{1y} = E_1 \sqrt{3}/2$

$I_{max} = (E_{1x} + E_2)^2 + E_{1y}^2 = \left[\left(10\tfrac{1}{2}\right)^2 + \tfrac{3}{4}\right] E_1^2 = 111 E_1^2$

$I_{min} = (E_x - E_2)^2 + E_{1y}^2 = \left[\left(9\tfrac{1}{2}\right)^2 + \tfrac{3}{4}\right] E_1^2 = 91 E_1^2$

$V = 0.099$

3



H ordinary, V extraordinary. $\theta_i = \alpha$

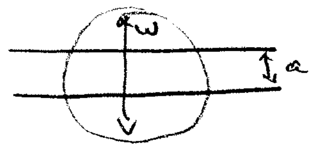
$\alpha_{cr}(H) = \sin^{-1}(1/n_o) = 37.3°$

$\alpha_{cr}(V) = \sin^{-1}(1/n_e) = 42.1°$

α must be between $\sin^{-1}\left(\frac{1}{n_o}\right)$ and $\sin^{-1}\left(\frac{1}{n_e}\right)$

H-polarization will be reflected.

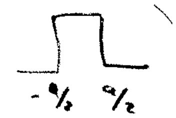
$\boxed{4}$



In the plane of the slit:

$$E(x\ y) = e^{-x^2/w^2}\, T_a(y)$$

(where T is the top-hat function



Variation of E in y-dimension neglected

Fourier transform: $\tilde{E}(k_x\ k_y) = e^{-\frac{k_x^2 w^2}{4}}\ \mathrm{sinc}\left(k_y \frac{a}{2}\right)$

In the far field: $x = \frac{k_x}{k} L$, $y = \frac{k_y}{k} L$

$$\tilde{E}(x\ y) = e^{-\frac{k^2 x^2 w^2}{4L^2}}\ \mathrm{sinc}\left(\frac{kya}{2L}\right)$$

$$I(xy) = e^{-\frac{k^2 x^2 w^2}{2L^2}}\ \mathrm{sinc}^2\left(\frac{kya}{2L}\right)$$

$\boxed{5}$ Pressure Δp_1 introduces a phase delay by half a period $\frac{T}{2} = \frac{\pi}{\omega}$

In order to make fringes disappear, we need a phase delay by $\tau_c = \frac{2\pi}{\Delta\omega}$ (coherence time)

$$\frac{\Delta p}{\tau_c} = \frac{\Delta p_1}{T/2} \Rightarrow \Delta p = 2\frac{\omega}{\Delta\omega}\Delta p_1 = 2\times 10^5 \text{ Pascal}$$

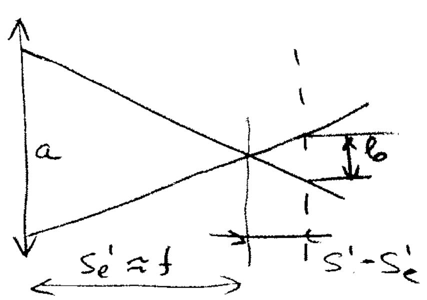
$\boxed{6}$ a) $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$

$$s' = \frac{sf}{s-f} = f\frac{1}{1-f/s} = f + f^2/s = 0.0525 \text{ m} \qquad (1)$$

b) $d = f\lambda/a = 5\cdot 10^{-2}\,\frac{5\times 10^{-7}}{10^{-2}} = 2.5\ \mu\text{m}$

c) For the object at e, $s'_e = f + f^2/e \overset{(1)}{=} s' + \frac{f^2}{e} - \frac{f^2}{s}$

$$= s' - f^2\frac{e-s}{es} \approx s' - f^2\frac{e-s}{s^2}$$



From similarity of triangles (fig.)

$$\frac{a}{f} = \frac{b}{s'-s'_e}$$

$$b = af\frac{e-s}{s^2}$$

Condition for $b < d$: $e - s < \frac{\lambda s^2}{a^2} = 5\text{ mm}$