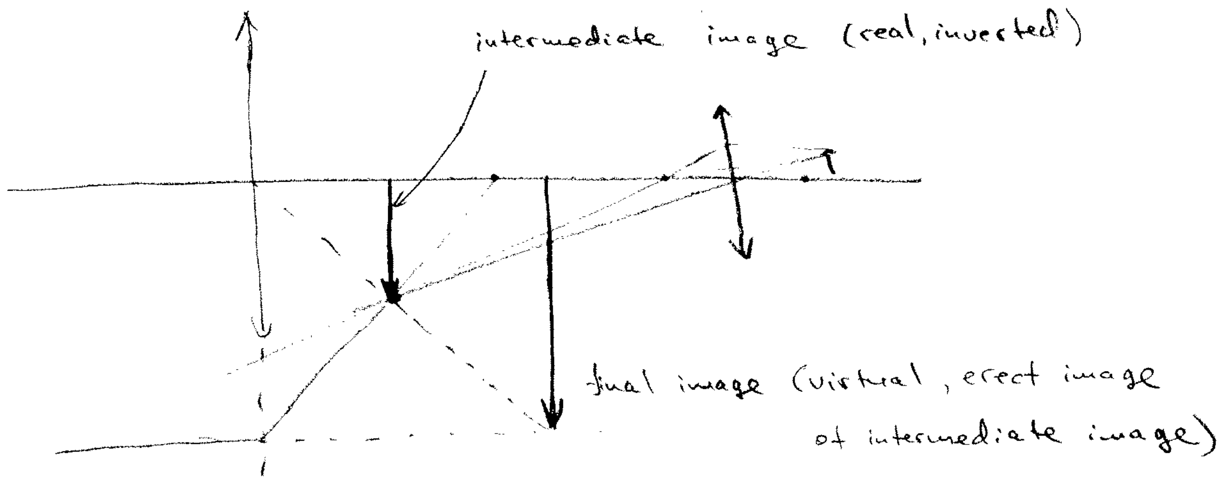
PHYS 471

Final examination

Solutions

$\boxed{1}$



Intermediate image: $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f_1}$

$$s' = \frac{f_1 s}{f_1 - s} = \frac{120}{12-10}\ mm = 60\ mm$$

(from the objective)

Distance of the intermediate image to the eyepiece:

$$s_i = L - s' = 40\ mm$$

Size of the intermediate image: $d_i = \frac{s'}{s} d = \frac{60}{12}\, 0.1\,mm = 0.5\,mm$

Final image: $-\frac{1}{s''} + \frac{1}{s_i} = \frac{1}{f_2}$

$$s'' = \frac{f_2 s_i}{f_2 - s_i} = \frac{2000}{50-40}\ mm = 200\ mm$$

(from the eyepiece)

Size of the final image:

$$d_f = \frac{s''}{s_i} d_i = \frac{200}{40}\, 0.5\,mm = 2.5\,mm$$

2 A $\lambda/2$ waveplate with optical axis at angle Θ will rotate linear polarization by angle 2Θ.

According to Malus's law, the transmitted intensity is then

$$I_t = I_0 \cos^2 2\Theta$$

$$I_r = I_0 \sin^2 2\Theta$$

3 The Jones matrix of the quarter-waveplate at $45°$ is

$$\hat{QWP} = 1 \cdot |+45°\rangle\langle+45°| + (-i)|-45°\rangle\langle-45°|$$

$$= \frac{1}{2}\begin{pmatrix}1\\1\end{pmatrix}\begin{pmatrix}1 & 1\end{pmatrix} + (-i)\frac{1}{2}\begin{pmatrix}1\\-1\end{pmatrix}\begin{pmatrix}1 & -1\end{pmatrix} = \frac{1}{2}\begin{pmatrix}1-i & 1+i\\1+i & 1-i\end{pmatrix}$$

Initially horizontal state

$$|H\rangle = \begin{pmatrix}1\\0\end{pmatrix}$$

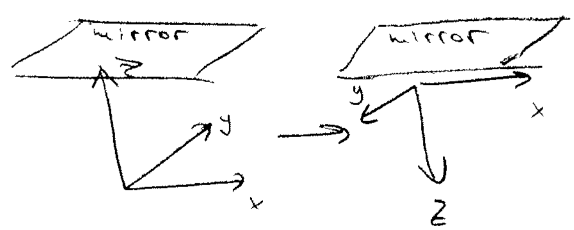
After QWP

phase factor neglect.

$$\hat{QWP}\,|H\rangle = \frac{1}{2}\begin{pmatrix}1-i\\1+i\end{pmatrix} = \frac{1-i}{\sqrt{2}}\frac{1}{\sqrt{2}}\begin{pmatrix}1\\i\end{pmatrix} = \frac{1-i}{\sqrt{2}}|R\rangle$$

On reflection, both horizontal and vertical polarization experience a phase shift by π. In addition, we flip the coordinate system because light must propagate along the z axis. This means either x-axis or y-axis must also change sign. Circularly polarized light will change handedness on reflection. We get

$$|L\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-i\end{pmatrix}$$



In order to study the effect of the QWP on the reflected light, we need to remember that the slow axis is now at $(-45°)$ with respect to this wave:

$$\hat{QWP}' = 1\,|+45°\rangle\langle-45°| + (+i)|-45°\rangle\langle-45°| = \frac{1}{2}\begin{pmatrix}1+i & 1-i\\1-i & 1+i\end{pmatrix}$$

So the output state is

phase factor.

$$\hat{QWP}'|L\rangle = \frac{1}{2\sqrt{2}}\begin{pmatrix}1+i-1-i\\1-i-i+1\end{pmatrix} = \frac{1}{2\sqrt{2}}\begin{pmatrix}0\\2-2i\end{pmatrix} = \frac{1-i}{\sqrt{2}}|V\rangle$$

4 $\frac{\Delta\lambda}{\lambda} \approx \frac{v}{c}$

Red light: ~ 600 nm

Green light: ~ 500 nm

$\frac{\Delta\lambda}{\lambda} \approx \frac{1}{5}$

$v = \frac{1}{5}c = 6 \cdot 10^7$ m/s $\approx 2 \cdot 10^8$ km/h (>> speed limit)

Fine: \$ $2 \cdot 10^9$

5 Beams 1 and 4: $I_{min,14} \propto (\sqrt{4} - \sqrt{1})^2 = 1$

$I_{max,14} \propto (\sqrt{4} + \sqrt{1})^2 = 9$

Beams 2 and 3: $I_{min,23} \propto (\sqrt{3} - \sqrt{2})^2 = 5 - 2\sqrt{6}$

$I_{max,23} \propto (\sqrt{3} + \sqrt{2})^2 = 5 + 2\sqrt{6}$

Because beams in each pair are in phase, the interference maxima and minima occur simultaneously for pairs (1-4) and (2-3). The total visibility is thus

$$V = \frac{(I_{max,14} + I_{max,23}) - (I_{min,14} + I_{min,23})}{(I_{max,14} + I_{max,23}) + (I_{max,14} + I_{max,23})}$$

$$= \frac{8 + 4\sqrt{6}}{10 + 10} = \frac{2 + \sqrt{6}}{5} = 0.89$$

6 Maximum angular resolution due to diffraction:

$\theta = \lambda / D = \lambda A / f = 0.5$ mrad (assuming $\lambda = 0.6\ \mu m$)

Optimal pixel size

$d = f\theta = \lambda A = 5\ \mu m$

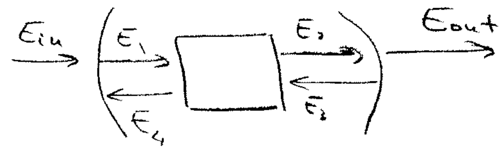
Total pixels

$\frac{ab}{d^2} = 2000 \times 1500 = 3$ Mpixels

$\boxed{7}$ Single-pass amplification:

in terms of intensity: $e^{\gamma L}$

in terms of amplitude: $e^{\gamma L/2}$



$$\begin{cases} E_1 = \sqrt{1-r^2}\, E_{in} + r\, E_4 \\ E_2 = e^{\frac{1}{2}\gamma L + i\varphi}\, E_1 \\ E_3 = r\, E_2 \\ E_4 = e^{\frac{1}{2}\gamma L + i\varphi}\, E_3 \end{cases}$$

$$E_1 = (1-r)E_{in} + r^2 e^{\gamma L} e^{2i\varphi} E_1$$

$$E_1 = E_{in}\, \frac{1-r}{1-r^2 e^{\gamma L} e^{2i\varphi}}$$

(φ is the phase shift due to one-way trip inside the cavity, $\varphi = \frac{2\pi\nu L}{c}$)

Transmitted amplitude:

$$E_{out} = E_2\sqrt{1-r^2} = E_{in}\, \frac{(1-r^2)\, e^{\frac{1}{2}\gamma L + i\varphi}}{1-r^2 e^{\gamma L} e^{2i\varphi}}$$

If we set $\tilde{r} = r e^{\frac{1}{2}\gamma L}$, we get

$$E_{out} = E_{in}\, \underbrace{\frac{(1-r^2)\, e^{\frac{1}{2}\gamma L}}{(1-\tilde{r}^2)}}_{\text{constant } (\equiv A)}\; \underbrace{\frac{(1-\tilde{r}^2)\, e^{i\varphi}}{1-\tilde{r}^2 e^{2i\varphi}}}_{\text{usual expression for FPI transmission}}$$

Intensity transmission (using the known expression for FPI)

$$T = \frac{|E_{out}|^2}{|E_{in}|^2} = A^2\, \frac{1}{1 + \frac{4\tilde{r}^2}{(1-\tilde{r}^2)^2}\sin^2\varphi}$$

$$T_{max} = A^2$$

$$T_{min} = A^2\, \frac{(1-\tilde{r}^2)^2}{(1+\tilde{r}^2)^2}$$

Finesse $\mathcal{F} = \frac{\pi \tilde{r}}{1-\tilde{r}^2}$

$\text{FWHM} = L/2c\mathcal{F}$

At $\gamma \to \gamma_{th} = \frac{2}{L}\ln\frac{1}{r}$, $\tilde{r} \to 1$, $A \to \infty$

$\Rightarrow T_{max} \to \infty$, $T_{min} \to 0$, $\mathcal{F} \to \infty$, $\text{FWHM} \to 0$.