University of Calgary
Winter semester 2007

PHYS 471: Optics

Homework assignment 2

Due February 5, 2007

Problem 2.1. A nearsighted person has a visual acuity of 20/300 (i.e. she/he can clearly see from a 20-foot distance letters with structural details that subtend one minute of arc at 300 feet). Estimate the optical power of prescription lenses this person requires.

Problem 2.2. A geometric image obtained with a camera obscura (pinhole camera) becomes clearer when the pinhole diameter is reduced. On the other hand, if the pinhole is too small, ray divergence due to diffraction ($\theta \approx \lambda/d$, where d is the pinhole diameter) will compromise the clarity. Estimate the diameter of the objective pinhole in a camera obscura of a 0.5 m depth at which the resolution is optimized.

Problem 2.3. Using the considerations similar to those in the previous problem, estimate the optimal size of the input slit of a visible light spectrometer which utilizes a prism made of flint glass with a 10 cm base (in this case, making the slit wider is beneficial because more light becomes available). To estimate the dispersion, use the following data: the index of refraction of flint glass equals 1.7076 at a wavelength of 656.3 nm (red) and 1.7328 at 486.1 nm (blue).

Problem 2.4. The fundamental atomic transition in rubidium has a wavelength of 795 nm. Assuming that rubidium vapor atoms in a glass cell obey the Maxwell distribution, find the shape of the absorption line (i.e. absorption as a function of the laser detuning from the center of the line, neglecting the overall coefficient). Estimate the line width (in Hz and nm). Estimate the size of a spectrometer prism necessary to resolve the shape of this line. The temperature is 300 K.

Problem 2.5. The refraction index of an atomic vapor cell in the neighborhood of an electromagnetically induced transparency (EIT) line is given by

$$n = 1 + \frac{\alpha_0 \lambda}{4\pi} \frac{\Gamma \Delta}{4\Delta^2 + \Gamma^2},$$

where $\alpha_0 \approx 1\ \mathrm{cm}^{-1}$, λ is the wavelength, $\Gamma = 10^4\ \mathrm{s}^{-1}$ is the EIT line width and $\Delta = \omega_{\mathrm{laser}} - \omega_0$ is the detuning of the laser frequency from the center of the EIT line.

a) Plot the index of refraction as a function of Δ.

b) Find the minimum and maximum value of the refraction index. Does the phase velocity become higher than the speed of light?

c) Find the group velocity at the center of the line.

d) Find the delay experienced by an optical pulse propagating through a 5 cm cell.

e) Find the group velocity for the case when EIT is replaced by electromagnetically-induced absorption, so the anomaly in the refraction index obtains the opposite sign. What is the interpretation of the observed phenomenon?

Feel free to approximate whenever reasonable.

Problem 2.6. Find the apex angle of a triangular flint glass prism such that a ray of wavelength 656.3 nm propagates through the prism in the minimum deviation configuration without losses. What polarization does the ray need to have? Find the intensity transmission coefficient for the orthogonal polarization. Do not make any significant approximations.

Problem 2.7. Re-derive the Fresnel equations for an interface of media with non-unitary magnetic permeabilities. **Hint:** it is more convenient to express the answer in terms of ϵ and μ than in terms of n.