University of Calgary
Winter semester 2007

PHYS 471: Optics

Homework assignment 6

Due April 13, 2007

Problem 6.1. A plane wave of wavelength $\lambda = 600$ nm is incident on a screen with a $d = 1$ mm slit. Calculate the intensity (in the units of unobstructed intensity) at the point located $L = 1$ m behind the screen at the border of the geometric image of the slit. Use numerical methods or the Cornu spiral pattern to calculate the integral.

Problem 6.2. A laser consists of a $L = 10$ cm Ng:YAG rod with the unsaturated gain $\gamma_0 = 0.05\ \text{cm}^{-1}$ and saturation intensity $I_S = 2500\ \text{W/cm}^2$ placed inside a ring cavity. The roundtrip intracavity losses (aside from the output coupler) are $1 - \eta = 0.01$. Find the optimum reflectivity of the output coupler and the output power of the laser with this output coupler if the $1/e^2$ radius beam of the beam inside the active medium is $w_0 = 1$ mm.

Problem 6.3. Propose and motivate a design (length, reflectivity) of an intracavity etalon for selecting a single mode of a He-Ne laser with a linear cavity of length $L = 50$ cm. The width of the gain profile is determined by the Doppler broadening.

Problem 6.4. The refraction indices of a fiber core and cladding are $n_1 = 1.49$ and $n_2 = 1.48$, respectively. What is the maximum diameter of the core at which the fiber remains single-mode for the wavelength $\lambda = 1\ \mu$m? What is the numerical aperture of such a fiber?

Problem 6.5. (extra credit) Consider a Gaussian pulse of electromagnetic field

$$E(t) = E_0 e^{-t^2/\tau^2} e^{-i\omega_0 t}$$

entering a single-mode fiber at time $t = 0$.

a) Calculate the spectrum $\tilde{E}(\omega, x = 0)$ of this pulse.

b) Calculate the spectrum $\tilde{E}(\omega, x = L)$ of this pulse after it has propagated distance L in the fiber. You will need to account for the phase accumulation due to material dispersion [the index of refraction of the core is $n_1(\omega)$]. Neglect waveguide dispersion and modal distortion.

c) Decompose the accumulated phase into a Taylor series around the point ω_0 up to the second order. Neglect higher order terms.

d) Perform an inverse Fourier transformation of $\tilde{E}(\omega, x = L)$ to obtain the time-dependent pulse amplitude. Verify that the zero- and first-order terms in the Taylor decomposition are associated with the phase and group velocities and do not affect the shape of the pulse.

e) Calculate the intensity of the propagated pulse as a function of time. Find the condition under which the broadening of the pulse is significant.