University of Calgary
Winter semester 2017

PHYS 443: Quantum Mechanics I

Midterm examination

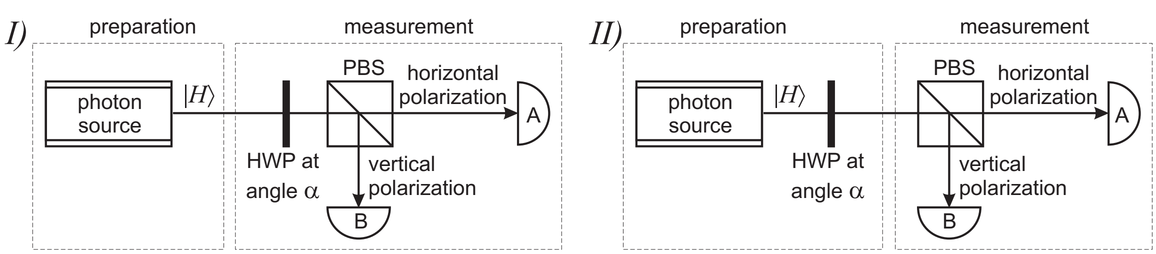
February 28, 2017, 20:00–22:00
Open books. No electronic equipment allowed.

Problem 1 (15 pts). A wave, initially linearly polarized at angle θ to horizontal, propagates through a quarter-wave plate with its optical axis oriented vertically. For $\theta = 0°, 30°, 60°, 90°$:

a) write the initial and final photon polarization states in the canonical basis;

b) sketch the initial and final polarization patterns. In each plot, mark the values of x and y at which the patterns cross the corresponding axes. Assume the initial amplitudes to follow the relation $A_x^2 + A_y^2 = 1$.

Problem 2 (25 pts).



The figure above shows two different interpretations of the same experiment. For both interpretations, perform the following calculations.

a) Determine the state of the photon entering the measurement part of the apparatus.

b) Determine the measurement basis.

c) Determine the probabilities of events in detectors A and B.

d) Events in detectors A and B are associated with the values $v_{A,B} = \pm 1$. Write the matrices of the corresponding observable in its eigenbasis and in the canonical basis.

e) Find the mean value of this observable in the depicted experiment.

Problem 3 (15 pts). Find the commutators

a) $[\hat{\sigma}_x + i\hat{\sigma}_y, \hat{\sigma}_x - i\hat{\sigma}_y]$;

b) $[\hat{\sigma}_x^2 + \hat{\sigma}_y^2 + \hat{\sigma}_z^2, \hat{\sigma}_z]$.

Hint: $[\hat{\sigma}_x, \hat{\sigma}_y] = 2i\hat{\sigma}_z$; $[\hat{\sigma}_y, \hat{\sigma}_z] = 2i\hat{\sigma}_x$; $[\hat{\sigma}_z, \hat{\sigma}_x] = 2i\hat{\sigma}_y$.

Problem 4 (20 pts). An atom has two energy eigenstates $|v_1\rangle, |v_2\rangle$ with the same energy eigenvalue E .

a) Write the matrix of the corresponding Hamiltonian $\hat{H}_0$.

b) At time $t = 0$ a field is turned on which makes the Hamiltonian equal to $\hat{H} = \hat{H}_0 + \hat{V}$ with $\hat{V} = E\,|v_1\rangle\langle v_2| + E\,|v_2\rangle\langle v_1|$. Write the matrix of the new Hamiltonian and of the associated evolution operator in the basis $\{|v_1\rangle, |v_2\rangle\}$.

c) At time $t = 0$, the atom is in state $|v_1\rangle$. Find the probability to find the atom in state $|v_2\rangle$ as a function of time.

Problem 5 (25 pts). Alice and Bob share the state

$$|\Psi\rangle = N(|HH\rangle + i\,|HV\rangle + 3\,|VV\rangle).$$

a) Find the normalization factor N.

b) Find $\hat{\sigma}_x \otimes \hat{\sigma}_z\,|\Psi\rangle$.

c) Alice performs a measurement on $|\Psi\rangle$ in the circularly polarized basis. What will be the probability of each possible result and the remotely prepared state of Bob's photon in each case?