University of Calgary
PHYS 443: Quantum Mechanics I

Polarization state measurement and tomography

Laboratory experiment

1 Purpose

- To understand the concepts of optical wave polarization;
- to become familiar with optical polarization instruments, such as the half- and quarter-wave plates and the polarizing beam splitter;
- to experimentally test the Measurement Postulate of quantum mechanics;
- to practice the application of operators to calculate transformations of quantum states;
- to experimentally reconstruct an unknown quantum state through measurements.

2 Equipment

- optical bench;
- 2 photodetectors connected to multimeters;
- helium-neon Laser;
- polarizing beam splitter (PBS);
- 2 half-wave plates (HWP);
- 2 quarter-wave plates (QWP)

3 Safety advice

Never look directly into the laser beam.

4 Equipment care and maintenance

Every optical element in this experiment costs several hundred dollars. Exercising proper care is essential for maintaining the experiment in a proper operational condition for future generations of students.

It is most important to avoid touching optical surfaces. If you do happen to accidentally touch a surface and leave a fingerprint, inform the lab manager *immediately* to have it cleaned. If you don't notice / forget / neglect to clean it, your body fat residue will enter a chemical reaction with the glass surface and damage it permanently after a few hours.

Another important advice is not to lay your optical elements flat on the table. Even a brief exposure to dust can permanently damage them. Whenever you remove an optic from the apparatus, place it in a holder or lay it down sideways, so the surfaces are vertical.

Do not forget to replace protective covers on your optics when you are leaving the lab.

5 Introductory note

This experiment deals with macroscopic quantities of light. All the results you are about to see can be explained by means of classical electromagnetic wave theory. However, because our goal is to understand quantum mechanics, this manual is written in terms of quantum polarization states of single photons. Because each photon in an electromagnetic wave has the same polarization state, macroscopic measurements on large numbers of photons, such as those performed here, reflect the quantum behavior of each individual photon.

The theory behind optical polarization states, quantum measurements and tomography is explained in Chapter 1 and Appendix C of the lecture notes.

6 Procedure

6.1 Preparation and alignment

a) Remove protective covers from the optics. Remember to replace them when finished.

b) Make sure the photodetectors are connected to power supplies and multimeters. Set the photodetectors' resistance to 3.3 kΩ. Make sure the power supplies and multimeters are turned on. Set the multimeters' limits to 2 Volts.

c) Turn on the laser. Align the laser beam parallel to the optical bench.

d) Place one of the photodetectors onto the optical bench about 30 cm in front of the laser. Align the detector so that the laser shines into the center of the detector's sensitive area.

   The multimeter's reading should be on the order of -1 Volt with the laser beam present and 0 when it is blocked (the negative sign is hereafter omitted). Move the photodetector in the transverse plane by ± 1 mm and check that the voltage reading does not change by more than a few per cent.

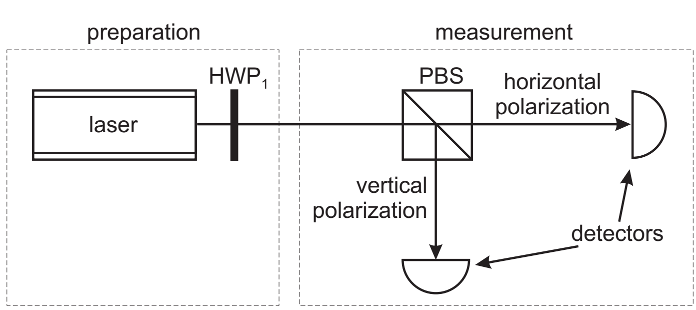
e) Place the PBS ~ 5 cm immediately before the detector. Because the laser is vertically polarized, the PBS should reflect the beam almost completely. Check that the voltmeter reading for the transmitted beam is less than 5% of the value measured without the PBS.

f) Place the second detector into the reflected beam. Make sure the beam hits the detector's sensitive area by repositioning the detector and/or tweaking the PBS mount screws. Check the reading on the corresponding multimeter to be on the order of 1 Volt. Move the photodetector in the transverse plane by ± 1 mm and check that the voltage reading does not change by more than a few per cent.

Try to avoid bumping the laser, PBS and detectors for the remainder of the experiment. Otherwise you may have to repeat the above alignment procedure.

6.2 Measuring linear polarization in the canonical basis

You have set up an apparatus for measuring the polarization in the canonical basis. Our next goal is to use this apparatus to measure various linearly polarized states.



a) Place one of the half-waveplates (HWP_1) in front ot the PBS. For consistency, *orient all waveplates so that the rotor of the waveplate frame is facing the laser.*

b) Rotate the HWP to minimize the transmission of the laser through the PBS. This means that the HWP is not changing the laser's polarization state, so the vertical laser polarization is either purely ordinary or purely extraordinary with respect to that waveplate. This means the waveplate's optic axis is either horizontal or vertical.

For the physics we are studying here, it does not matter whether the orientation of the optic axis is horizontal or vertical. Indeed, if the matrix of the half-waveplate operator with a horizontal optic axis in the canonical basis is $\hat{A}_{\text{HWP}}(0) \simeq \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$, then the same waveplate with the optic axis oriented vertically it is $\hat{A}_{\text{HWP}}(90°) \simeq \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. The photon states obtained through the action of these two operators are different only by the unphysical overall phase factor -1. As a matter of convention, we shall assume the optic axis in the present position of the waveplate to be horizontal.

Record the angle reading β_{HWP_1} on the waveplate frame. Although the optic axis is horizontal, this reading may not equal zero because the waveplate is glued to the frame at a random angle. For any other reading β, the angle between the waveplate's axis and the horizontal is $\alpha = \beta - \beta_{\text{HWP}_1}$.

Tips:

- Write down the number on the sticker attached to the waveplate.
- Do not use any markings on the wavelpate frame to determite the optic axis angle. These markings are not reliable.

c) Record the voltage $V_{r,\max}$ of the multimeter in the reflected channel.

Then rotate the waveplate by about 45° to turn the polarization to horizontal. Make a fine adjustment to the waveplate to minimize the reflection from the PBS. Record the voltage $V_{t,\max}$ of the multimeter in the transmitted channel.

We shall refer to the procedure of measuring $V_{t,\max}$ and $V_{r,\max}$ as *calibrating the detectors*. Keep in mind that the calibration values may drift during the measurement run. You may wish to re-calibrate your detectors both before and after each series of data, and perhaps even a few times during the data run. Also, the detectors should be re-calibrated for every data acquisition run with a different set of waveplates because the waveplates are partially absorptive.

d) Rotate the waveplate to prepare a set of linearly polarized states $|\theta\rangle$. The waveplate fill "flip" the vertically polarized input photon about its optic axis. If the optic axis is at the angle $\alpha = \beta - \beta_{\text{HWP}_1}$ to horizontal, you will produce a photon linearly polarized at $\theta = 2\alpha - 90°$ with respect to horizontal. For each θ, measure the voltage in the transmitted and reflected channels, V_t and V_r, respectively, as a function of the waveplate angle β. These voltages translate into photon detection probabilities according to $\text{pr}_t = V_t/V_{t,\max}$ and $\text{pr}_r = V_r/V_{r,\max}$.

Record your data into Table 1. Plot the experimentally acquired pr_t and pr_r as a function of θ together with theoretical curves:

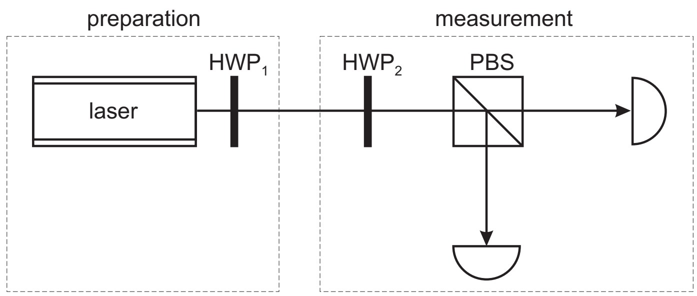
$$\text{pr}_t = |\langle H|\,\theta\rangle|^2 = \left|\begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}\right|^2 = \cos^2\theta;$$

$$\text{pr}_r = |\langle V|\,\theta\rangle|^2 = \left|\begin{pmatrix} 0 & 1 \end{pmatrix}\begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}\right|^2 = \sin^2\theta.$$

The last column in the table is for self-check. If you get a value that is significantly different from 1, you are doing something wrong; probably the detectors are not correctly calibrated.

A typical error observed in these experiments should not exceed a few per cent. If the behavior you see is dramatically different from the theoretically expected, stop and think!

6.3 Measuring linear polarization in the diagonal basis



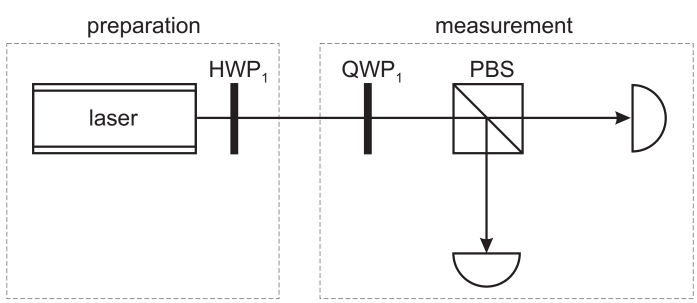
a) Remove HWP_1. Place another half-waveplate (HWP_2) in front ot the PBS. Record the angle β_{HWP_2} for which no polarization rotation is taking place. Again, we shall assume that this angle corresponds to the optic axis being horizontal.

b) Rotate the waveplate by angle 22.5°. In this position, the waveplate will transform photon polarization states according to $|{+45^\circ}\rangle \to |H\rangle$, $|{-45^\circ}\rangle \to |V\rangle$, thereby facilitating measurement in the diagonal basis.

c) Reinsert HWP_1 in front of HWP_2. Similarly to the previous subsection, use HWP_1 to prepare different linear polarization states $|\theta\rangle$. Perform measurements of these states in the diagonal basis. Acquire the data into Table 1. Plot the experimentally acquired pr_t and pr_r as a function of θ together with theoretical curves: $\mathrm{pr}_t = |\langle +45^\circ|\,\theta\rangle|^2$ and $\mathrm{pr}_r = |\langle -45^\circ|\,\theta\rangle|^2$. This time, please derive the theoretical expression independently.

In order to calibrate the detectors, you can temporarily set HWP_1 to angle β_{HWP_1} and HWP_2 to β_{HWP_2} or $\beta_{\mathrm{HWP}_2}+45^\circ$. In this way, the first waveplate will not affect the laser's vertical polarization and the second one will either keep it vertical or rotate to horizontal. You may wish to adjust the waveplates a bit in order to minimize the amount of light going to the other detector.

6.4 Measuring linear polarization in the circular basis



a) Remove both HWPs. Place a quarter-waveplate (QWP_1) in front ot the PBS. Record the angle β_{QWP_1} for which no polarization transformation is taking place. Again, let us assume that this angle corresponds to the horizontal orientation of the optic axis (in the case of quarter-wave plates, there is a subtlety that we shall address later, but for now, let us use this assumption).

b) Rotate the QWP by 45°. In this position, the waveplate will transform photon polarization states according to $|L\rangle \to |H\rangle$, $|R\rangle \to |V\rangle$, thereby facilitating measurement in the circular basis. A simple argument to that effect is given in Ex. 1.11 in the lecture notes; a more rigorous calculation can be performed using the result of Ex. 1.24[1].

c) Reinsert HWP_1 in front of QWP_1. Similarly to the previous subsection, use HWP_1 to prepare different linear polarization states $|\theta\rangle$. Perform measurements of these states in the circular basis. Acquire the data into Table 1. Compare the theoretical and experimental plots.

In order to calibrate the detectors, you can temporarily turn QWP_1 to angle β_{QWP_1} and HWP_1 to β_{HWP_1} or $\beta_{\mathrm{HWP}_1}+45°$. HWP_1 will then either keep the laser field vertically polarized or rotate it to horizontal. In both cases, the QWP with the horizontal optic axis will not affect the polarization state.

6.5 Matching the quarter-wave plate axes

Our subsequent experiments require us to address an important ambiguity associated with determining the angle of the quarter-wave plate axes. The operators (1.5b) associated with the optic axis oriented horizontally and vertically are

$$\hat{A}_{\mathrm{QWP}}(0) \simeq \begin{pmatrix} i & 0 \\ 0 & 1 \end{pmatrix}$$

and

$$\hat{A}_{\mathrm{QWP}}(90°) \simeq \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} = i \begin{pmatrix} -i & 0 \\ 0 & 1 \end{pmatrix},$$

respectively. These operators, in contrast to the HWP, are no longer related by phase factors. we can see from the above equations that the waveplate in these two positions, when applied to a linearly polarized photon, will generate states with opposite helicity. Such states are physically different, and this difference can be seen by measurement.

To see the difference, let us suppose we have another QWP with the optical axis that is *known* to be horizontal. Let us position this second waveplate after the first one and transmit a 45° polarized photon through both of them. If the optic axis of the first waveplate is horizontal, we obtain

$$\hat{A}_{\mathrm{QWP}}(0)\hat{A}_{\mathrm{QWP}}(0)\,|+\rangle \simeq \frac{1}{\sqrt{2}}\begin{pmatrix} i & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} i & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} -1 \\ 1 \end{pmatrix} \simeq -\,|-\rangle\,,$$

whereas if it is vertical, we have

$$\hat{A}_{\mathrm{QWP}}(90°)\hat{A}_{\mathrm{QWP}}(0)\,|+\rangle \simeq \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}\begin{pmatrix} i & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} i \\ i \end{pmatrix} \simeq i\,|+\rangle\,.$$

These two states have orthogonal linear polarization.

This result is easily understood intuitively. The two QWPs with their axes being parallel form a HWP. This HWP will transform $|+45°\rangle$ into $|-45°\rangle$. On the other hand, two QWPs with orthogonal optic axes will cancel each other's action, so the photon state will remain unchanged.

In our next experiment, two QWPs will be used. In accordance with the above discussion, it is necessary that their optic axes be defined in a consistent manner. To this end, perform the following procedure.

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[1]Indeed, substituting $\beta = 45°$ into Eq. (1.5b), we find

$$A_{QWP} \simeq \frac{1}{2}\begin{pmatrix} i+1 & i-1 \\ i-1 & i+1 \end{pmatrix} = \frac{1}{\sqrt{2}}\frac{i+1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$$

and hence

$$A_{QWP}\,|L\rangle \simeq \frac{1}{2}\frac{i+1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}\begin{pmatrix} 1 \\ -i \end{pmatrix} = \frac{i+1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \simeq \frac{i+1}{\sqrt{2}}\,|H\rangle\,;$$

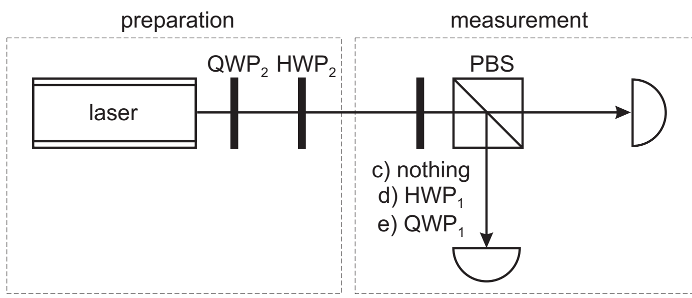
$$A_{QWP}\,|R\rangle \simeq \frac{1}{2}\frac{i+1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}\begin{pmatrix} 1 \\ i \end{pmatrix} = \frac{i+1}{\sqrt{2}}\begin{pmatrix} 0 \\ i \end{pmatrix} \simeq i\frac{i+1}{\sqrt{2}}\,|V\rangle\,.$$

a) Remove all waveplates from the setup and insert another quarter-wave plate, QWP_2. Record the angle β_{HWP_2} such that QWP_2 makes no change to the laser polarization.

b) Insert QWP_1 either in front of or behind QWP_2. Turn both waveplates to angles $\beta_{\mathrm{HWP}_1}+45^\circ$ and $\beta_{\mathrm{HWP}_2}+45^\circ$, respectively. Check the polarization of the transmitted light. If the polarization is horizontal, the two QWPs together act as a HWP, so β_{HWP_2} is recorded correctly. If it is vertical, add 90° to β_{HWP_2} and record the value as the new β_{HWP_2}. Repeat the procedure to verify that the two waveplates' axes are now parallel.

Note that the above procedure does not determine the direction of the waveplates' optic axes; it only ensures that they are defined consistently with each other. This is however sufficient for the purposes of our experiment.

6.6 Measuring an arbitrary polarization state and quantum tomography



a) Let us now prepare an arbitrary polarization state. Remove all waveplates from the apparatus. Then reinsert QWP_2 in the position closest to the laser and set it to the angle $\beta_{\mathrm{QWP}_2}+\alpha_Q$, where α_Q is equal to your birth date plus ten degrees. For example, if you were born on September 11, $\alpha_Q = 21^\circ$.

b) Insert HWP_2 after QWP_2 and set it to angle $\beta_{\mathrm{HWP}_2}+\alpha_H$ that is equal to twice your birth month plus ten degrees. For September 11, set $\alpha_H = 28^\circ$.

c) Perform the polarization measurement in the canonical basis by recording V_t and V_r. Calibrate the detector by temporarily setting $\alpha_Q = 0$ and $\alpha_H = 0^\circ, 45^\circ$.

d) Perform the polarization measurement in the diagonal basis. Insert HWP_1 after both QWP_2 and HWP_2, but before the PBS. Set HWP_1 to $\beta_{\mathrm{HWP}_1}+22.5^\circ$ and record V_t and V_r. Calibrate the detector by setting both QWP_1 and HWP_1 to β_{QWP_1} and β_{HWP_1}, respectively (so they don't change the horizontally or vertically polarized light); then set HWP_2 to angles β_{HWP_2} or $\beta_{\mathrm{HWP}_2}+45^\circ$ to make all the light vertically or horizontally polarized, respectively. Adjust the waveplates slightly to minimize the emission to the other detector.

e) Perform the polarization measurement in the circular basis. Replace HWP_1 by QWP_1. Set QWP_1 to $\beta_{\mathrm{QWP}_1}+45^\circ$ and record V_t and V_r. Calibrate the detector in the same manner as above.

f) Record the data into Table 2 and calculate the respective experimental and theoretical values for the probabilities $\mathrm{pr}_{H,V,+,-,R,L}$.

For the theoretical calculation, use the waveplate operators to determine the transformation of the vertically polarized photons that the laser emits by the two waveplates:

$$\begin{aligned}|\psi\rangle &= \hat{A}_{\mathrm{HWP}}(\alpha_H)\hat{A}_{\mathrm{QWP}}(\alpha_Q)\,|V\rangle \\ &\simeq \begin{pmatrix} -\cos 2\alpha_H & -\sin 2\alpha_H \\ -\sin 2\alpha_H & \cos 2\alpha_H \end{pmatrix}\begin{pmatrix} \sin^2\alpha_Q + i\cos^2\alpha_Q & (i-1)\sin\alpha_Q\cos\alpha_Q \\ (i-1)\sin\alpha_Q\cos\alpha_Q & i\sin^2\alpha_Q+\cos^2\alpha_Q \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix};\end{aligned}$$

then calculate the expected probability according to the Measurement Postulate of quantum mechanics. Verify consistency between the theory and experiment. In the event of significant inconsistency, repeat the measurements and calculations. Do include the values of α_H, α_Q as well as the calculated matrices of $\hat{A}_{\rm HWP}(\alpha_H)$, $\hat{A}_{\rm QWP}(\alpha_Q)$ and $|\psi\rangle$ in your report.

g) Perform quantum state tomography based on your measured probabilities. First, notice that, by multiplying by a phase factor, any qubit state $|\psi\rangle$ can be brought to the form $|\psi'\rangle \simeq \begin{pmatrix} a_H \\ a_V e^{i\phi} \end{pmatrix}$, where a_H and a_V are real and positive, ϕ is real and $a_H^2 + a_V^2 = 1$. Our goal is to find a_H, a_V and ϕ. This can be done, for example, as follows.

- Notice that

$$\text{pr}_H = \left| \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} a_H \\ a_V e^{i\phi} \end{pmatrix} \right|^2 = a_H^2$$

  (because a_H is real and positive). The values of pr_H and pr_V are therefore sufficient to find $a_{H,V}$.

- We also have

$$\text{pr}_+ = \frac{1}{2} \left| \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} a_H \\ a_V e^{i\phi} \end{pmatrix} \right|^2 = \frac{1}{2}(a_H^2 + a_V^2 + 2a_H a_V \cos\phi) = \frac{1}{2} + a_H a_V \cos\phi;$$

$$\text{pr}_R = \frac{1}{2} \left| \begin{pmatrix} 1 & -i \end{pmatrix} \begin{pmatrix} a_H \\ a_V e^{i\phi} \end{pmatrix} \right|^2 = \frac{1}{2}(a_H^2 + a_V^2 + 2a_H a_V \sin\phi) = \frac{1}{2} + a_H a_V \sin\phi.$$

With the knowledge of a_H, these equations can be unambiguously solved to find ϕ. For best precision, determine this angle using

$$\tan\phi = \left(\text{pr}_L - \frac{1}{2}\right) \Big/ \left(\text{pr}_+ - \frac{1}{2}\right)$$

and adjust the inverse tangent by $\pm\pi$ based on your knowledge of the signs of $\cos\phi$ and $\sin\phi$.

After you have found a_H a_V and ϕ, verify that your $|\psi\rangle$ and $|\psi'\rangle$ can be related by a phase factor. Note that in the calculation of $|\psi'\rangle$ may be significantly affected by the accumulation of experimental errors, so the discrepancy may be higher than a few per cent expected in the remainder of the experiment.

h) Use software to plot the polarization patterns associated with the theoretically expected state as well as the state reconstructed by quantum tomography.

7 What to include in the lab report

- For Sec. 6.2–6.4:
  - theoretical calculations of the expected probabilities;
  - tables containing all data;
  - theoretical and experimental plots (superimposed) for the probabilities pr_t, pr_r as functions of the state's polarization angle θ.
- For Sec. 6.6:
  - theoretical calculation of the expected state;
  - tables containing all data;
  - calculation for the state tomography reconstruction;
  - polarization pattern plots for the theoretically expected and reconstructed state.

Table 1: a suggested template for recording measurement data for parts 6.2–6.4.

$\beta(^\circ)$	$\beta_{\mathrm{HWP}_1}(^\circ)$	$\alpha(^\circ)$	$\theta(^\circ)$	V_t(V)	$V_{t,\max}$(V)	pr_t	V_r(V)	$V_{r,\max}$(V)	pr_r	$\mathrm{pr}_t + \mathrm{pr}_r$
		0								
		10								
		20								
		30								
		40								
		50								
		60								
		70								
		80								
		90								

$\beta(^\circ)$	$\beta_{\mathrm{HWP}_1}(^\circ)$	$\alpha(^\circ)$	$\theta(^\circ)$	V_t(V)	$V_{t,\max}$(V)	pr_t	V_r(V)	$V_{r,\max}$(V)	pr_r	$\mathrm{pr}_t + \mathrm{pr}_r$
		0								
		10								
		20								
		30								
		40								
		50								
		60								
		70								
		80								
		90								

$\beta(^\circ)$	$\beta_{\mathrm{HWP}_1}(^\circ)$	$\alpha(^\circ)$	$\theta(^\circ)$	V_t(V)	$V_{t,\max}$(V)	pr_t	V_r(V)	$V_{r,\max}$(V)	pr_r	$\mathrm{pr}_t + \mathrm{pr}_r$
		0								
		10								
		20								
		30								
		40								
		50								
		60								
		70								
		80								
		90								

Table 2: A suggested template for recording measurement data for part 6.6. Abbreviation "th." stands for "theoretical"

Canonical basis

V_t	$V_{t,\max}$	$\mathrm{pr}_H = \mathrm{pr}_t$	th. pr_H	V_r	$V_{r,\max}$	$\mathrm{pr}_V = \mathrm{pr}_r$	th. pr_V	$\mathrm{pr}_H + \mathrm{pr}_V$

Diagonal basis

V_t	$V_{t,\max}$	$\mathrm{pr}_+ = \mathrm{pr}_t$	th. pr_+	V_r	$V_{r,\max}$	$\mathrm{pr}_- = \mathrm{pr}_r$	th. pr_-	$\mathrm{pr}_+ + \mathrm{pr}_-$

Circular basis

V_t	$V_{t,\max}$	$\mathrm{pr}_L = \mathrm{pr}_t$	th. pr_L	V_r	$V_{r,\max}$	$\mathrm{pr}_R = \mathrm{pr}_r$	th. pr_R	$\mathrm{pr}_L + \mathrm{pr}_R$