University of Calgary
Winter semester 2017

PHYS 443: Quantum Mechanics I

Homework assignment 4

Due March 14, 2017

Problem 4.1. A Bell inequality test is performed as described in the lecture notes, but with single-photon detectors of non-unity efficiency η. Alice's or Bob's measurement apparata are so constructed that in the event neither detector in the apparatus has clicked, the displayed value will be randomly $+1$ or -1 with equal probabilities. What is the η value range for which the Bell inequality is violated?

Problem 4.2. The c-not gate in the canonical basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ (where the first qubit is control second target) has the matrix

$$\hat{U}_{\text{c-not}} = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & & 1 \\ & & 1 & \end{pmatrix}.$$

Assuming that this matrix corresponds to the evolution of the qubit under some Hamiltonian $\hat{H}$ for the time t, find $\hat{H}$[1].

Problem 4.3. The quantum teleportation protocol is implemented with state $|\Phi^-\rangle$ as the entangled resource, instead of $|\Psi^-\rangle$. Verify that the protocol will still work. Determine the local operations that Bob will need to perform in order to obtain a copy of Alice's state in the event of each outcome of Alice's Bell measurement.

Problem 4.4. In the quantum repeater described in Ex. 2.67 in the lecture notes, the Bell measurement that is performed on photon pairs within each link is only able to detect states $|\Psi^\pm\rangle$, but not $|\Phi^\pm\rangle$. On the other hand, the Bell measurement on memory cells, which is used to connect the links, is perfect. Find the time t required to obtain entanglement between Alice's and Bob's memory cells with a probability of at least 1/2.

Problem 4.5. For two functions $f(x)$ and $g(x)$, $\int\limits_{-\infty}^{+\infty} f^*(x)g(x)\mathrm{d}x = A$. Find $\int\limits_{-\infty}^{+\infty} \tilde{f}^*(k)\tilde{g}(k)\mathrm{d}k$, where $\tilde{f}(k)$ and $\tilde{g}(k)$ are the Fourier transforms.

Problem 4.6. Find the direct and inverse Fourier transforms of the following functions (with $\kappa, a, b > 0$).

a) $f(x) = e^{-\kappa|x|}$.

b) $f(x) = e^{-\kappa|x|}\sin x$.

c) $f(x) = e^{ik_0(x-a)-(x-a)^2/2b^2}$.

d) $f(x) = x\sin x$. (**Hint:** use the same trick as when calculating the Fourier transform of a derivative).

[1] Even though the problem has many solutions, a single solution suffices for full credit.