University of Calgary
Winter semester 2017

PHYS 443: Quantum Mechanics I

Homework assignment 1

Due January 24, 2017

Problem 1.1. Consider the set S consisting of the following three vectors: $\vec{w}_1 = (0, 2, 1)$, $\vec{w}_2 = (1, 2, 0)$, $\vec{w}_3 = (1, 0, 1)$.

a) Decompose an arbitrary vector $\vec{a} = (x, y, z)$ as a linear combination of elements of S.

b) Show that S is a basis in the three-dimensional geometric vector space.

c) Perform the Gram-Schmidt procedure for this set. Verify that the basis obtained in this procedure is indeed orthonormal.

Problem 1.2. Two states are decomposed in the circular basis according to

$$|\psi\rangle = \frac{1}{5}(3i\,|R\rangle + 4\,|L\rangle), \quad |\phi\rangle = \frac{1}{5}(4i\,|R\rangle - 3\,|L\rangle), \tag{1}$$

a) Show that these states form an orthonormal basis using the fact that the circular basis is orthonormal.

b) Find the decompositions of these states in the canonical basis by finding the matrices of $|\psi\rangle$, $|\phi\rangle$, $|H\rangle$ and $|V\rangle$ in the circular basis and using the inner product rule (A.6) from the lecture notes. Write your answer both in the Dirac and matrix notations.

c) Verify your answer for part (b) by expressing $|R\rangle$ and $|L\rangle$ in the canonical basis and substituting into Eq. (1);

d) Verify that states $|\psi\rangle$ and $|\phi\rangle$ form an orthonormal set using the inner product in the canonical basis.

e) Repeat parts (b)–(d) for the diagonal basis rather than canonical.

f) Decompose states $|H\rangle, |V\rangle, |R\rangle, |L\rangle, (|H\rangle + 2i\,|V\rangle)/\sqrt{5}$ in basis $\{|\psi\rangle, |\phi\rangle\}$. Write your answer both in the Dirac and matrix notations.

g) States $|H\rangle, |V\rangle, |R\rangle, |L\rangle, (|H\rangle + 2i\,|V\rangle)/\sqrt{5}$ are measured in basis $\{|\psi\rangle, |\phi\rangle\}$. What are the probabilities of the outcomes?

Problem 1.3 A classical electromagnetic wave with amplitude $\sqrt{A_H^2 + A_V^2} = 1$ V/m and phases $\phi_H = \phi_V$ is polarized at angle 30° to horizontal. The operation $\phi_V \to \phi_V + \Delta\phi$ is applied to this wave. Describe and plot the polarization patterns for $\phi = 0, \pi/2, \pi, 3\pi/2, \ldots$.

Problem 1.4 States

a) $|\psi\rangle = \sqrt{\frac{1}{4}}\,|H\rangle + \sqrt{\frac{3}{4}}\,|V\rangle$,

b) $|\psi\rangle = \sqrt{\frac{1}{4}}\,|H\rangle - \sqrt{\frac{3}{4}}\,|V\rangle$,

c) A statistical mixture of either $|H\rangle$ with probability $\frac{1}{4}$ or $|V\rangle$ with probability $\frac{3}{4}$

are measured in the diagonal basis. Find the probabilities of the measurement outcomes.

Problem 1.5 Consider the modified BB84 protocol in which Alice sends and Bob analyzes the photon in the following polarization bases: $\{|H\rangle, |V\rangle\}$ and $\{|\theta\rangle, \left|\frac{\pi}{2}+\theta\right\rangle\}$ (this protocol becomes BB84 for $\theta = 45^\circ$). Angle θ is known to Alice, Bob and Eve. Find the bit error rate that Alice and Bob will see in the event of a straightforward "intercept-resend" attack, in which Eve intercepts the photon, measures it in one of the above two bases (randomly chosen with equal probabilities), and resends whatever she detected. There are no losses, all equipment is perfect.