University of Calgary
Winter semester 2017

PHYS 443: Quantum Mechanics I

Final examination

April 22, 2017, 8:00–11:00
Open books. No electronic equipment allowed.

Problem 1 (15 pts) A particle of energy E is incident from the left on a potential structure

$$V(x) = \begin{cases} 0, & x < 0 \\ V_1, & 0 \le x \le a \\ V_2, & x \ge a \end{cases}$$

Write the set of equations for calculating the wavefunction of the energy eigenstate for all x with

a) $V_1 < 0 < E < V_2$;

b) $0 < V_2 < V_1 < E$.

Sketch the real part of the wavefunction for both cases, capturing its salient features.

Problem 2 (15 pts). Write the matrices of the rescaled position and momentum observables of a harmonic oscillator in the Fock basis. For full credit, show the 5×5 top left portions of the two matrices.

Problem 3 (20 pts). State $|\psi\rangle$ has the wavefunction

$$\psi(x) = \begin{cases} \mathcal{N}x, & 0 \le x \le a \\ 0, & x < 0 \text{ and } x > a \end{cases}$$

in the position basis, with a real positive a. Find

a) the normalization factor $\mathcal{N}$;

b) the mean and variance of the position observable;

c) the wavefunction in the momentum basis.

Hint: When calculating the Fourier transform, use the formula for the derivative of an exponential.

Problem 4 (20 pts). A qubit, initially in the state $|\psi(0)\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, evolves under the Hamiltonian $\hat{H} = \hbar\omega\hat{\sigma}_y$.

a) Find the matrix of the evolution operator.

b) Find the quantum mean value of the observable $\hat{\sigma}_z$ as a function of time t, calculating the evolution in the Schrödinger picture.

c) Find the quantum mean value of the observable $\hat{\sigma}_z$ as a function of time t, calculating the evolution in the Heisenberg picture.

Problem 5 (30 pts). Alice's single photon and Bob's mechanical oscillator are in the entangled state

$$|\Psi\rangle = \mathcal{N}(|H\rangle \otimes |\alpha\rangle + |V\rangle \otimes |-\alpha\rangle),$$

where $|\alpha\rangle$ is a coherent state with a real eigenvalue α.

a) Find the normalization factor $\mathcal{N}$.

b) Alice performs a polarization measurement on her photon in the basis $\{|\theta\rangle, \left|\frac{\pi}{2}+\theta\right\rangle\}$ and Bob performs a measurement on his oscillator in the Fock basis. Alice and Bob do not communicate. Find the probability of each possible measurement result for Alice and Bob.

c) Alice instead performs a joint Bell measurement on her photon and another photon in the state $a|H\rangle + b|V\rangle$ with $|a|^2 + |b|^2 = 1$. Find the probabilities of occurrence and the resulting state of Bob's oscillator for each of the four possible Alice's results. Under which conditions would you classify this experiment as proper quantum teleportation?