University of Calgary
Winter semester 2014

PHYS 443: Quantum Mechanics I

Midterm examination

February 26, 2015, 18:30

Open books. No electronic equipment allowed.
Full credit = 100 points. Attempt all problems. Partial credit will be given.

Problem 1 (10). Observables $\hat{X}$ and $\hat{P}$ have commutator $[\hat{X}, \hat{P}] = i$. Find $[X^2 + P^2, X + iP]$.

Problem 2 (15). Measurements of observable $\hat{A}$ in state $|H\rangle$ yield results 0 and 1 with probabilities 1/2 each. Measurements of observable $\hat{B}$ in state $|H\rangle$ yields result 2 with probability 3/4 and result 4 with probability 1/4. It is also known that $[\hat{A}, \hat{B}] = ix\hat{\sigma}_z$. Find the upper bound on the absolute value of x.

Problem 3 (15). The initial polarization state of a photon in the canonical basis is given by $\frac{1}{\sqrt{10}}\begin{pmatrix} 1 \\ 3i \end{pmatrix}$. For each of the operations below applied to that state, draw and classify the resulting polarization pattern.

a) no operation;

b) a half-wave plate with the optical axis oriented at angle 0;

c) a half-wave plate with the optical axis oriented at angle $\pi/6$;

d) a quarter-wave plate with the optical axis oriented at angle 0.

Your plots must show the orientation of the polarization ellipses' major semiaxises and the lengths of both semiaxes.

Problem 4 (20). *Schmaser components*™ manufactures a waveplate which imposes phase difference φ upon the ordinary and extraordinary polarization waves, where φ is as requested by the customer. This waveplate is inserted into a photon's path with its optical axis oriented at angle θ to horizontal. Answer the following questions for any given φ and θ.

a) What are the eigenstates of the operator $\hat{U}$ of the polarization transformation enacted by this waveplate? How are they transformed by the waveplate?

b) Write this operator in the matrix form in its eigenbasis. Write the corresponding expression in the Dirac notation.

c) Write the matrix of $\hat{U}$ in the canonical basis.

d) Suppose operator $\hat{U}$ corresponds to the evolution under some Hamiltonian $\hat{H}$ for time t_0. Find the matrix of this Hamiltonian in its eigenbasis as well as the corresponding expression in the Dirac notation. What are the energy eigenvalues?

Problem 5 (15). Alice and Bob share two photons in state

$$|\Psi\rangle = \mathcal{N}(|HH\rangle + 2\,|HV\rangle - 3\,|VH\rangle + 4i\,|VV\rangle).$$

Alice performs a local measurement in the diagonal basis.

a) what is the normalization factor $\mathcal{N}$?

b) What are the probabilities of possible outcomes?

c) What (normalized) state will Bob's photon be remotely prepared in after the measurement in the case of each possible outcome?

Problem 6 (25). An atom is described in some basis $\{|v_1\rangle, |v_2\rangle\}$ by Hamiltonian

$$\hat{H} = \hbar\omega \begin{pmatrix} 1 & 3i \\ -3i & 9 \end{pmatrix}.$$

a) Find the energy eigenstates and eigenvalues.

b) Energy is measured in state $|\psi_0\rangle = \frac{1}{\sqrt{2}}(|v_1\rangle + i\,|v_2\rangle)$. Find the probabilities to detect each energy eigenvalue, as well as the mean and variance of this measurement.

c) The atom is initially in state $|\psi_0\rangle$. Find its state $|\psi(t)\rangle$ at arbitrary moment t.