University of Calgary
Winter semester 2015

PHYS 443: Quantum Mechanics I

Homework assignment 3

Due February 24, 2015 strictly before 11:00am

Problem 3.1. Alice and Bob share two photons in polarization state

$$|\Psi\rangle = \frac{1}{\sqrt{11}}(|HH\rangle + i\,|VH\rangle + 3\,|VV\rangle).$$

a) Alice and Bob both perform measurements on their respective photons. Find the probabilities of all possible results.

b) Only Bob performs a polarization measurement on his photon. Find the probability of each outcome and the remotely prepared state of Bob's photon after the measurement. Apply each of the two alternative techniques to solve the problem in each basis:

- using the partial inner product;
- decomposing the initial state according to

$$|\Psi\rangle = \sum_i \frac{1}{N_i}\,|a_i\rangle \otimes |w_i\rangle\,,$$

where $\{|w_i\rangle\}$ is Bob's measurement basis and $\{|a_i\rangle\}$ is a set of normalized states in Alice's Hilbert space (see Sec. 2.2.1 of the lecture notes).

c) Verify that the probability values found in parts (a) and (b) are consistent with each other.

Solve the above problem for all measurements performed in (i) canonical and (ii) circular bases.

Problem 3.2. Let us define observable

$$\hat{\sigma}_\theta = |\theta\rangle\langle\theta| - \left|\frac{\pi}{2}+\theta\right\rangle\left\langle\frac{\pi}{2}+\theta\right|$$

(so that $\hat{\sigma}_{\theta=0} = \hat{\sigma}_z$ and $\hat{\sigma}_{\theta=\pi/4} = \hat{\sigma}_x$).

a) Find the matrix of $\hat{\sigma}_\theta$ in the canonical basis. **Hint:** denote $c = \cos 2\theta$, $s = \sin 2\theta$.

b) For observable $\hat{\sigma}_z \otimes \hat{\sigma}_\theta$:

i) calculate the matrix in the canonical basis $\{|HH\rangle\,, |HV\rangle\,, |VH\rangle\,, |VV\rangle\}$;

ii) determine the eigenstates and eigenvalues (**hint:** you need not solve any equations);

iii) calculate the expectation value and uncertainty in Bell state $|\Psi^-\rangle$.

Problem 3.3. An atom has two energy eigenstates $|v_1\rangle\,, |v_2\rangle$ with eigenvalues 0 and $3\hbar\omega$, respectively, with $\omega > 0$.

a) Write the matrix of the corresponding Hamiltonian $\hat{H}_0$.

b) At time $t = 0$ a field is turned on which makes the Hamiltonian equal to $\hat{H} = \hat{H}_0 + \hat{V}$ with $\hat{V} = 2i\hbar\omega\,|v_1\rangle\langle v_2| - 2i\hbar\omega\,|v_2\rangle\langle v_1|$. Write the matrix of the new Hamiltonian and of the associated evolution operator in basis $\{|v_1\rangle\,, |v_2\rangle\}$.

c) At time $t = 0$, the atom is in state $|v_1\rangle$. Find all values of time t at which the the probability to find the atom in state $|v_2\rangle$ is maximized.

Problem 3.4. Two qubits interact according to the Hamiltonian

$$\hat{H} = \hbar\omega\hat{\sigma}_x \otimes \hat{\sigma}_x.$$

The initial state of the qubits is $|\Psi(0)\rangle = |\Phi^+\rangle$. Find $|\Psi(t)\rangle$ in the canonical basis.