University of Calgary
Winter semester 2015

PHYS 443: Quantum Mechanics I

Homework assignment 6

Due April 14, 2015 in SB 318

Problem 6.1. The *single-photon added coherent states* (SPACS) are obtained from coherent states by action of the creation operator: $|\alpha, 1\rangle = \mathcal{N}\hat{a}^\dagger |\alpha\rangle$.

a) Find normalization factor $\mathcal{N}$.

b) Find the decomposition of this state in the photon number basis (you are not required to simplify the result).

c) Find the expectation value of the position observable.

d) Find the wavefunction of the SPACS for a real α.

e) Which quantum state does SPACS approach in the limit $\alpha = 0$? $\alpha \to \infty$?

Problem 6.2. End-of-chapter Problem 3.9 from the lecture notes.
Note a typo in Eqs. (3.87) in the lecture notes. The correct set of equations should read

$$\text{Transmission: } \frac{j_E}{j_A} = \left|\frac{E}{A}\right|^2 = \frac{4k_0^2k_1^2}{4k_0^2k_1^2 + (k_1^2+k_0^2)^2\sinh^2(k_1L)}; \tag{1}$$

$$\text{Reflection: } \frac{j_B}{j_A} = \left|\frac{B}{A}\right|^2 = \frac{(k_1^2+k_0^2)^2\sinh^2(k_1L)}{4k_0^2k_1^2 + (k_1^2+k_0^2)^2\sinh^2(k_1L)}. \tag{2}$$

Problem 6.3. At time $t = 0$, state $|\psi\rangle$ of a harmonic oscillator has mean position $\langle X\rangle = X_0$, mean momentum $\langle P\rangle = P_0$. Calculate the mean values of the position and momentum observables of this state as a function of time.

Problem 6.4. From expressions (4.16) in the lecture notes for the angular momentum components in spherical coordinates, derive these components in Cartesian coordinates (4.11).

Problem 6.5.

a) Find the matrices of $\hat{L}_x$, $\hat{L}_y$, $\hat{L}_z$, $\hat{L}_\pm$, and $\hat{L}^2$ explicitly for $l = 3/2$.

b) Verify that these matrices obey $\hat{L}_x^2 + \hat{L}_y^2 + \hat{L}_z^2 = \hat{L}^2$.

c) For these matrices, determine the commutators $[\hat{L}_i, \hat{L}_j]$ and verify that they are consistent with the known commutation relations for the angular momentum components.

Problem 6.6. For state $|n, n-1, m\rangle$ the hydrogen atom,

a) calculate the radial component $R_{n,n-1}(r)$;

b) calculate $\langle r\rangle$.

Problem 6.7. Calculate $\langle 100|\, \hat{x}\, |211\rangle$ for the hydrogen atom.