University of Calgary
Winter semester 2015

PHYS 443: Quantum Mechanics I

Homework assignment 4

Due March 10, 2015 at 9:30am

Problem 4.1. A Bell inequality test, as described in the lecture notes, is performed with a defective entangled source which produces a statistical mixture of state $|\Psi^-\rangle$ with probability η and $|\Psi^+\rangle$ with probability $1-\eta$. What is the range of η values for which the Bell inequality is violated?

Problem 4.2. The quantum teleportation protocol is implemented with state $|\Phi^+\rangle$ as the entangled resource, instead of $|\Psi^-\rangle$. Verify that the protocol will still work. Determine the local operations that Bob will need to perform in order to obtain a copy of Alice's state in the event of each outcome of Alice's Bell measurement.

Problem 4.3. For two functions $f(x)$ and $g(x)$, $\int_{-\infty}^{+\infty} f^*(x)g(x)\mathrm{d}x = A$. Find $\int_{-\infty}^{+\infty} \tilde{f}^*(k)\tilde{g}(k)\mathrm{d}k$, where $\tilde{f}(k)$ and $\tilde{g}(k)$ are the Fourier transforms.

Problem 4.4. Find the Fourier transform of the following functions (with $\kappa, a > 0$).

a) $f(x) = e^{-\kappa|x|}$.

b) $f(x) = e^{-\kappa|x|}\cos x$.

c) $f(x) = e^{ik_0 x-(x-a)^2/2b^2}$.

d) $f(x) = x\cos x$. (**Hint:** use the same trick as when calculating the Fourier transform of a derivative).

Problem 4.5. A state has wavefunction

$$\psi(x) = Axe^{-\kappa^2 x^2/2}.$$

a) Find the normalization factor A.

b) Find the wavefunction $\tilde{\psi}(p)$ in the momentum basis.

c) Verify the uncertainty principle: $\langle \Delta p^2 \rangle \langle \Delta x^2 \rangle \geq \hbar^2/4$.

Hint:

$$\int_{-\infty}^{+\infty} x^2 e^{-x^2} dx = \frac{\sqrt{\pi}}{2}; \quad \int_{-\infty}^{+\infty} x^4 e^{-x^2} dx = \frac{3\sqrt{\pi}}{4}.$$

Problem 4.6. Find the matrix element $\left\langle p \middle| \hat{A} \middle| p' \right\rangle$ if operator $\hat{A}$ is a function of position: $\hat{A} = e^{-\hat{x}^2/b^2}$.