PHYS 443: Quantum Mechanics I

Homework assignment 5

Due March 24, 2015 at 9:30am

Problem 5.1. An eigenstate $|\psi\rangle$ of the Hamiltonian with potential $V(x)$ has wavefunction

$$\psi(x) = \frac{1}{(\pi d^2)^{1/4}} e^{-\frac{x^2}{2d^2}}. \tag{1}$$

a) Is this a bound state? Do there exist energy eigenstates with lower energies?

b) Find $V(x)$ and the corresponding energy eigenstate. Let $V(0) = 0$.

c) At time $t = 0$, the potential instantly switches to $V'(x) = V(2x)$.

   i) Find the wavefunction $\psi'(x)$ of the corresponding energy eigenstate in the new potential.

   ii) What is the probability that after the switching the particle will be found in that state?

Problem 5.2. Recalling that $\mathrm{pr}(x) = \psi(x)\psi^*(x)$, derive the continuity equation:

$$\frac{d\,\mathrm{pr}(x)}{dt} = -\frac{dj}{dx}, \tag{2}$$

where

$$j = -i\frac{\hbar}{2m}\left(\psi^*\frac{d\psi}{dx} - \psi\frac{d\psi^*}{dx}\right) \tag{3}$$

is the probability density current.

Problem 5.3. End-of-chapter Problem 3.6 from the lecture notes.

Problem 5.4. End-of-chapter Problem 3.9 from the lecture notes.
   Note a typo in Eqs. (3.87) in the lecture notes. The correct set of equations should read

$$\text{Transmission: } \frac{j_E}{j_A} = \left|\frac{E}{A}\right|^2 = \frac{4k_0^2 k_1^2}{4k_0^2 k_1^2 + (k_1^2 + k_0^2)^2 \sinh^2(k_1 L)}; \tag{4}$$

$$\text{Reflection: } \frac{j_B}{j_A} = \left|\frac{B}{A}\right|^2 = \frac{(k_1^2 + k_0^2)^2 \sinh^2(k_1 L)}{4k_0^2 k_1^2 + (k_1^2 + k_0^2)^2 \sinh^2(k_1 L)}. \tag{5}$$