University of Calgary
Winter semester 2015

PHYS 443: Quantum Mechanics I

Final examination

April 24, 2015, 8:00–11:00 (3 hours)

Open books. No electronic equipment allowed.
Full credit = 100 points. Attempt all problems. Partial credit will be given.

Problem 1 (10). Find the commutator $[\hat{L}_x\hat{L}_y, \hat{L}_z^2]$.

Problem 2 (15). Consider a massive particle of mass m attached to a spring with spring constant k. The other end of the spring is attached to a wall, resulting in harmonic oscillatory motion.

a) Write the full set of energy eigenvalues and the corresponding normalized wavefunctions in the *non-rescaled* position basis.

b) Suppose another wall is inserted at point $x = 0$ as shown in Fig. 1, so the particle cannot go into region $x > 0$. How should the above set be modified in order to represent the energy eigenvalues and eigenstates for the new potential?

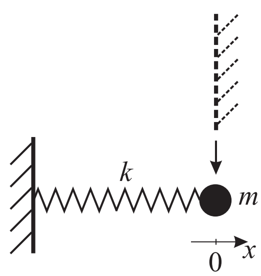

Figure 1: illustration to Problem 2

Problem 3 (25). A photon (with Alice) and an atom (with Bob) are in an entangled state

$$|\Psi\rangle = \frac{1}{\sqrt{6}}\left[(|H\rangle + |V\rangle) \otimes |1,1\rangle + 2\,|V\rangle \otimes |1,-1\rangle\right],$$

where the atomic states are written in the $|lm\rangle$ basis. Alice measures her photon's polarization in the diagonal basis.

a) Determine the states in which the atom will be prepared in the event of each possible outcome of Alice's measurement, as well as the probability of each outcome.

b) Find the mean and uncertainty of the angular momentum component $\hat{L}_x$ for the atom for each of these states.

c) Suppose Alice does not perform a measurement or the result of her measurement is unknown. Find the mean and uncertainty of the angular momentum component $\hat{L}_x$ for the atom.

Problem 4 (25). For the potential shown in Fig. 2:

a) write the general solution $\psi(x)$ of the time-independent Schrödinger equation for each x if the energy E is known to be between 0 and V_0;

b) write all relevant wavefunction continuity relations and express them as conditions on the amplitudes of the solution found in part (a);

c) sketch qualitatively the wavefunction of the ground state. For full credit, your plot must clearly show the behavior of the wavefunction in different spatial regions, as well as at boundaries between them.

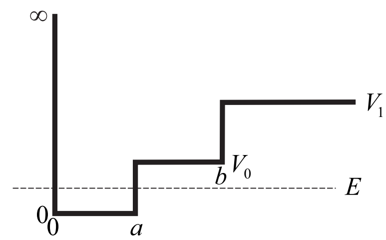

Figure 2: illustration to Problem 4

Problem 5 (25). An electron in an atom is initially in a common eigenstate $|\psi(0)\rangle$ of operators $\hat{L}^2$ and $\hat{L}_x$ with orbital quantum number $l = 1$ and the eigenvalue of $\hat{L}_x$ equal to $\hbar$.

a) What is the eigenvalue of $\hat{L}^2$ in this state?

b) Express $|\psi(0)\rangle$ in the matrix notation in the $|lm\rangle$ basis.

c) Write the wavefunction $\psi_0(\theta, \phi)$ of $|\psi(0)\rangle$ explicitly.

d) The system evolves under the Hamiltonian

$$\hat{H} = \hbar\omega(|1,1\rangle\langle 1,0| + |1,0\rangle\langle 1,1|).$$

Write the matrix of this Hamiltonian and find the corresponding evolution opeartor.

e) Find the evolution $|\psi(t)\rangle$ of the electron's state.

When solving this problem, restrict your calculations to the subspace of the Hilbert space corresponding to $l = 1$.

Angular momentum component matrices for $l = 1$ in the $|lm\rangle$ basis:

$$\hat{L}_x \simeq \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \hat{L}_y \simeq \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \hat{L}_z \simeq \hbar \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$