Phys 443

Second midterm examination

Solutions

1

$$|\psi_1\rangle \otimes |\Psi_{23}\rangle = (3\alpha|HHH\rangle + 4i\alpha|HVV\rangle + 3\beta|VHH\rangle + 4i\beta|VVV\rangle)/5$$

$$= \frac{1}{5\sqrt{2}}\Big(3\alpha(|\Phi^+\rangle + |\Phi^-\rangle)|H\rangle + 4i\alpha(|\Psi^+\rangle + |\Psi^-\rangle)|V\rangle$$
$$+ 3\beta(|\Psi^+\rangle - |\Psi^-\rangle)|H\rangle + 4i\beta(|\Phi^+\rangle - |\Phi^-\rangle)|V\rangle$$

Project onto $|\Psi^-_{12}\rangle$

$$|\psi_{out}\rangle = \langle\Psi^-_{12}|\big(|\psi_1\rangle \otimes |\Psi_{23}\rangle\big) = (4i\alpha|V\rangle - 3\beta|H\rangle)/5\sqrt{2}$$

Probability $\quad \langle\psi_{out}|\psi_{out}\rangle = \dfrac{16|\alpha|^2 + 9|\beta|^2}{50}$

2

$$\int|\tilde{\psi}(p)|^2 dp = \int \psi^*(p)\psi(p)\,dp =$$

$$= \frac{1}{2\pi\hbar}\iiint \psi^*(x)e^{i\frac{px}{\hbar}}\psi(x')e^{-i\frac{px'}{\hbar}}\,dp\,dx\,dx'$$

$$= \frac{1}{2\pi\hbar}\iint \psi^*(x)\psi(x')\int e^{i\frac{p(x-x')}{\hbar}}\,dp\,dx\,dx'$$

$$= \frac{1}{2\pi\hbar}\iint \psi^*(x)\psi(x')\,2\pi\,\delta\!\left(\frac{x-x'}{\hbar}\right)dx\,dx'$$

$$= \frac{1}{2\pi\hbar}\iint \psi^*(x)\psi(x')\,2\pi\hbar\,\delta(x-x')\,dx\,dx'$$

$$= \int \psi^*(x)\psi(x)\,dx$$

3

$$\langle x|p^2|\psi\rangle = \langle x|p\,p|\psi\rangle = \int\langle x|p|x'\rangle\langle x'|p|\psi\rangle\,dx'$$

$$= \int(-i\hbar)\frac{d}{dx}\delta(x-x')(-i\hbar)\frac{d}{dx'}\psi(x')\,dx'$$

$$= -\hbar^2\frac{d}{dx}\frac{d}{dx}\psi(x) = -\hbar^2\frac{d^2}{dx^2}\psi(x)$$