University of Calgary
Winter semester 2011

PHYS 443: Quantum Mechanics I

Second midterm examination

March 29, 2011

Open books. Attempt all questions. Partial credit will be given.

Problem 1 (40 pts). Alice and Bob implement a communication protocol that is analogous to quantum teleportation except that the entangled state they share is $|\Psi_{23}\rangle = (3\,|HH\rangle + 4i\,|VV\rangle)/5$. Initial Alice's state is $|\psi_1\rangle = \alpha H + \beta V$ (with $|\alpha|^2 + |\beta|^2 = 1$).

a) Find the state prepared at Bob's location when Alice detects state $|\Psi^-\rangle$ in her Bell measurement.

b) Find the probability of this event.

Problem 2 (30 pts). A certain quantum state has wavefunction $\psi(x)$ in the position basis. The state is normalized, such that

$$\int_{-\infty}^{+\infty} |\psi(x)|^2\, dx = 1. \tag{1}$$

As we discussed in class, the wavefunction of this state in the momentum basis equals

$$\tilde{\psi}(p) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{+\infty} \psi(x) e^{-i\frac{px}{\hbar}}\, dx. \tag{2}$$

Using Eqs. (1) and (2), show explicitly that

$$\int_{-\infty}^{+\infty} |\tilde{\psi}(p)|^2\, dp = 1. \tag{3}$$

Problem 3 (30 pts). A certain quantum state $|\psi\rangle$ has wavefunction $\psi(x)$ in the position basis. What is the wavefunction of the state $\hat{p}^2\,|\psi\rangle$ in the position basis? For credit, your answer must be derived in the Dirac notation.
Hint: $\langle x|\,\hat{p}\,|x'\rangle = -i\hbar\frac{d}{dx}\delta(x-x')$**;** $\langle x|\,\hat{p}\,|\psi\rangle = -i\hbar\frac{d}{dx}\psi(x)$**.**

Good luck!