Phys 443

First midterm examination

Solutions

1 a) $\hat{H} = (3|v_1\rangle\langle v_1| + 1|v_1\rangle\langle v_2| + 1|v_2\rangle\langle v_1| + 3|v_2\rangle\langle v_2|)E_0$

b) $\langle v_1|\hat{H}|v_1\rangle = 3E_0$

c) $\begin{vmatrix} 3E_0-\lambda & 1 \\ 1 & 3E_0-\lambda \end{vmatrix} = 0$

$\lambda^2 - 6\lambda E_0 + 8E_0^2 = 0$

$\lambda_1 = E_1 = 2E_0$ Find eigenvector $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 0 \Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = |E_1\rangle$

$\lambda_2 = E_2 = 4E_0$ $\begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 0 \Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = E_2$

d) $\hat{H} = 2E_0|E_1\rangle\langle E_1| + 4E_0|E_2\rangle\langle E_2|$

e) $e^{i\Theta H} = e^{2i\Theta E_0}|E_1\rangle\langle E_1| + e^{4i\Theta E_0}|E_2\rangle\langle E_2|$

$$= \frac{1}{2}\begin{pmatrix} e^{2i\Theta E_0} + e^{4i\Theta E_0} & -e^{2i\Theta E_0} + e^{4i\Theta E_0} \\ -e^{2i\Theta E_0} + e^{4i\Theta E_0} & e^{2i\Theta E_0} + e^{4i\Theta E_0} \end{pmatrix}$$

f) $\hat{U} = e^{-i\frac{\hat{H}}{\hbar}t}$

$$\hat{U}|v_1\rangle = \frac{1}{2}\begin{pmatrix} e^{-2i\frac{E_0}{\hbar}t} + e^{-4i\frac{E_0}{\hbar}t} \\ e^{-2i\frac{E_0}{\hbar}t} - e^{-4i\frac{E_0}{\hbar}t} \end{pmatrix}$$

$pr_1 = |\langle v_1|\hat{U}|v_1\rangle|^2 = 1$ if $\langle v_2|\hat{U}|v_1\rangle = 0$.

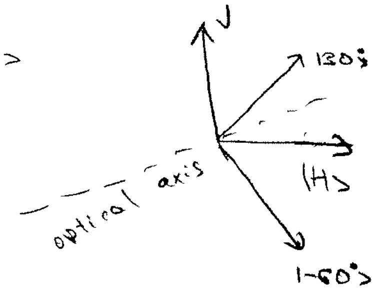
(second component of $\hat{U}|v_1\rangle$ is zero)

$$e^{-2i\frac{E_0}{\hbar}t} = 1$$

$$2i\frac{E_0}{\hbar}t = 2\pi$$

$$t = \frac{\pi\hbar}{E_0}$$

$\boxed{2}$ a) $A|\hat{H}\rangle = |30^\circ\rangle = \cos 30^\circ |H\rangle + \sin 30^\circ |V\rangle = \frac{\sqrt{3}}{2}|H\rangle + \frac{1}{2}|V\rangle$



b) $A|\hat{V}\rangle = |-60^\circ\rangle = \frac{1}{2}|H\rangle - \frac{\sqrt{3}}{2}|V\rangle$

c) $\hat{A}_{ij} = \langle v_i|\hat{A}|v_j\rangle = \begin{pmatrix} \sqrt{3}/2 & 1/2 \\ 1/2 & -\sqrt{3}/2 \end{pmatrix}$

d) $\hat{A}\hat{A}^{+} = \hat{A}\hat{A} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

e) $\hat{A}_{ij}|R\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} \sqrt{3}/2 & 1/2 \\ 1/2 & -\sqrt{3}/2 \end{pmatrix}\begin{pmatrix} 1 \\ i \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\sqrt{3}+i}{2} \\ \frac{1-\sqrt{3}i}{2} \end{pmatrix}$

$$= \frac{1}{\sqrt{2}}\frac{\sqrt{3}+i}{2}\begin{pmatrix} 1 \\ \frac{1-\sqrt{3}i}{\sqrt{3}+i} \end{pmatrix} = \frac{1}{\sqrt{2}}\frac{\sqrt{3}+i}{2}\begin{pmatrix} 1 \\ \frac{(\sqrt{3}-i)(1-\sqrt{3}i)}{4} \end{pmatrix}$$

$$= \frac{1}{\sqrt{2}}\frac{\sqrt{3}+i}{2}\begin{pmatrix} 1 \\ i \end{pmatrix} = \underbrace{\frac{\sqrt{3}+i}{2}}_{\uparrow\text{phase factor}}|R\rangle$$