University of Calgary
Winter semester 2007

PHYS 443: Quantum Mechanics I
First midterm examination

March 1, 2011

Open books. Attempt all questions. Partial credit will be given.

Problem 1. Consider an atom whose Hamiltionian in some orthonormal basis $\{|v_1\rangle, |v_2\rangle\}$ has the following matrix:

$$\hat{H} = E_0 \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}.$$

a) (5 pts) Write the Hamiltonian in the Dirac notation in terms of states $|v_1\rangle, |v_2\rangle$.

b) (10 pts) The energy observable is measured in the state $|v_1\rangle$. What is the expectation value of the measured energy?

c) (20 pts) Find the stationary states $|E_1\rangle, |E_2\rangle$ of this Hamiltonian and the corresponding energy values.

d) (5 pts) Express the Hamiltonian in the Dirac notation in terms of the energy eigenstates.

e) (10 pts) Find $e^{i\theta\hat{H}}$, where θ is a real number. Express your answer in the matrix form, in the basis $\{|v_1\rangle, |v_2\rangle\}$ (simplifying the answer is not required).

f) (extra credit — 20 pts) The atom is initially in state $|v_1\rangle$. How much time will elapse until the next time the atom's state becomes $|v_1\rangle$ again (up to a phase factor)?

Problem 2.

a) (10 pts) A horizontally polarized photon propagates through a half-wave plate oriented at $15°$ to horizontal. Write the new state of the photon in the Dirac notation in the canonical basis.

b) (10 pts) Answer the same question for the initial photon of vertical polarization.

c) (10 pts) Use the answers from parts (a) and (b) to find the operator matrix of the transformation associated with this waveplate. Write this matrix in the canonical basis.

d) (10 pts) Verify that this operator is unitary.

e) (10 pts) Using the result of part (c), find how the right circular polarization state is transformed by this waveplate.

Good luck!