University of Calgary
Winter semester 2011

PHYS 443: Quantum Mechanics I

Homework assignment 6

Due April 14, 2011

Problem 6.1. A particle is in the ground state of an infinite potential box. The left wall of the box is located at $x = -a/2$, the right wall at $x = a/2$. Suddenly the box shrinks in size. Find the probability of finding the particle in the ground state of the new potential if

a) the walls reposition themselves to $x = \pm a/4$;

b) only the left wall repositions itself to $x = 0$.

Problem 6.2. Consider a particle of mass m, whose initial state has wavefunction $\psi(x)$, in an infinite potential box of width a. Show that the evolution under the Schrödinger equation will restore the initial state (possibly with a phase factor) after time $T = 4ma^2/\pi\hbar$.

Problem 6.3. Find the bound energy spectrum of the potential that contains two delta-function wells: $V(x) = -V_0\delta(x - a/2) - V_0\delta(x + a/2)$ under the assumption that the wells are located very far away from each other. Find and plot the associated stationary states. Verify that your result is consistent with that for a single well in the limit of infinite a.

Problem 6.4. For the barrier potential,

$$V(x) = \begin{cases} 0 \text{ for } x \leq 0 \text{ or } x > L \\ V_0 \text{ for } 0 < x \leq L \end{cases},$$

a) Find the solution of the time-independent Schrödinger equation corresponding to a de Broglie wave entering from the left and $E > V_0$.

b) Find the transmission and reflection coefficients for the probability current. Is their sum equal to one?

Problem 6.5. The *single-photon added coherent states* (SPACS) are obtained from coherent states by action of the creation operator: $|\alpha, 1\rangle = \hat{a}^\dagger |\alpha\rangle$.

a) Find the decomposition of this state in the photon number basis.

b) Find the expectation values and uncertainties of the position and momentum observables.

c) Find the wavefunction of the SPACS for a real α.

d) Which quantum state does SPACS approach in the limit $\alpha = 0$? $\alpha \to \infty$?

Problem 6.6 *(to be confirmed).*

a) Find the matrices of $\hat{L}_x$, $\hat{L}_y$, $\hat{L}_z$, $\hat{L}_\pm$, and $\hat{L}^2$ explicitly for $l = 1$.

b) Verify that these matrices obey $\hat{L}_x^2 + \hat{L}_y^2 + \hat{L}_z^2 = \hat{L}^2$.

c) For these matrices, determine the commutators $[\hat{L}_i, \hat{L}_j]$, $[\hat{L}_z, \hat{L}_\pm]$ and $[\hat{L}_+, \hat{L}_-]$ and verify that they are consistent with the general commutation relations for the angular momentum derived in class.

Problem 6.7 *(to be confirmed).* Spin-1/2 particles are prepared in eigenstates of the component of the angular momentum vector operator which points in the direction between the x and z axes at angle θ to the z axis. The spin is measured with the Stern-Gerlach apparatus whose field gradient is directed along the z axis. In what fractions will the particles split?