University of Calgary
Winter semester 2011

PHYS 443: Quantum Mechanics I

Homework assignment 1

Due January 25, 2011

Problem 1.1.

a) Using the applet at `http://www.amanogawa.com/archive/Polarization/Polarization-2.html`, generate states $|+30^\circ\rangle$, $|-60^\circ\rangle$, $|R\rangle$, $|L\rangle$. Record the values of amplitudes and phases used in each case. Verify that these values are consistent with Definition A.1 (note that the applet's definitions of the left and right circular polarizations are opposite to ours).

b) Download the demonstration at `http://demonstrations.wolfram.com/PolarizationOfAnOpticalWaveThroughPolarizersAndWavePlates/` (to run the demo, if you don't have Mathematica, you will also need the Mathematica Player at `http://www.wolfram.com/products/player/download.cgi`). Verify that the transformations of the waves under the action of waveplates is consistent with that described in Section A.3 of the lecture notes. No written reporting is required for this part of the homework.

Problem 1.2.

a) Decompose the linear polarization states $|\theta\rangle$ and $\left|\frac{\pi}{2}+\theta\right\rangle$ (where θ is an arbitrary angle) into the canonical basis. Write the decomposition in the matrix form.

b) Show that these states form an orthonormal basis.

c) Decompose the left circular polarization state $|L\rangle$ into this basis. Discuss the physics behind the observed dependence on θ.

d) Decompose the state $|\psi\rangle = (i\,|H\rangle + 4\,|V\rangle)/\sqrt{17}$ in this basis.

e) Find the inner product $\langle\psi|L\rangle$ using the "row-times-column" rule both in the canonical basis and in the basis $\{|\theta\rangle, \left|\frac{\pi}{2}+\theta\right\rangle\}$. Show that the results are identical.

Problem 1.3.

a) Propose an experimental apparatus for measuring the polarization of the photon in the basis $\{|\theta\rangle, \left|\frac{\pi}{2}+\theta\right\rangle\}$.

b) What are the probabilities of each outcome if this apparatus is used to measure a photon with a linear polarization at angle $|\varphi\rangle$ to horizontal?

c) The same question for photons with circular polarizations.

Problem 1.4. Verify that the following three vectors: $\vec{w}_1 = (1,0,1)$, $\vec{w}_2 = (1,2,0)$, $\vec{w}_3 = (0,-1,0)$ form a basis in the three-dimensional geometrical space. Perform the Gram-Schmidt procedure for this set. Verify that the basis obtained in this procedure is indeed orthonormal.