University of Calgary
Winter semester 2011

PHYS 443: Quantum Mechanics I

Homework assignment 4

Due March 15, 2011

Problem 4.1. Perform the following calculations for operator $\hat{A} = \hat{\sigma}_y \otimes \hat{\sigma}_y$ in the biqubit Hilbert space.

a) Write the matrix of $\hat{A}$ in the canonical basis $\{|HH\rangle, |HV\rangle, |VH\rangle, |VV\rangle\}$.

b) Write the matrix of $\hat{A}$ in the Bell basis $\{|\Phi^+\rangle, |\Phi^-\rangle, |\Psi^+\rangle, |\Psi^-\rangle\}$.

c) Determine the eigenstates and eigenvalues of $\hat{A}$. **Hint:** you need not solve any equations.

d) Find the expectation value of observable $\hat{A}$ in state $|\Theta\rangle = (3|HH\rangle + 4|VV\rangle)/5$.

Problem 4.2. Consider the state $|\Theta\rangle = (4|HH\rangle + 3i|VV\rangle)/5$ shared between Alice and Bob.

a) Alice performs a measurement on $|\Theta\rangle$ in the canonical basis. Using the Second Postulate (extension to multipartite measurements), find the probabilities $\mathrm{pr}_{\mathrm{Alice},H}$ and $\mathrm{pr}_{\mathrm{Alice},V}$ of the two possible measurement results. What state will be remotely prepared in Bob's Hilbert space in both cases?

b) Both Alice and Bob measure $|\Theta\rangle$ in the canonical basis. Using the original Second postulate, find the probabilities pr_{HH}, pr_{HV}, pr_{VH}, pr_{VV} of the four outcomes.

c) Verify that the results of parts (a) and (b) are consistent with each other, i.e. $\mathrm{pr}_{\mathrm{Alice},H} = \mathrm{pr}_{HH} + \mathrm{pr}_{HV}$ and $\mathrm{pr}_{\mathrm{Alice},V} = \mathrm{pr}_{VH} + \mathrm{pr}_{VV}$.

d) Repeat parts (a)–(c) for measurements in the circular basis.

Problem 4.3. Consider the *Greenberger-Horne-Zeilinger* state $|\Psi_{GHZ}\rangle = \frac{1}{\sqrt{2}}(|HHH\rangle + |VVV\rangle)$ distributed among Alice, Bob, and Charley.

a) Alice and Bob perform a joint measurement on $|\Psi_{GHZ}\rangle$. What is the probability for them to detect

- $|\Psi^-\rangle$,
- $|HR\rangle$,
- $|\Theta\rangle$ from Problem 4.2.

and onto which state will Charley's particle project? For each of the above states, assume any measurement basis that contains the state in question.

b) Rewrite state $|\Psi_{GHZ}\rangle$

- in the Bell basis in the Hilbert space of Alice and Bob and canonical in Charley's Hilbert space;
- in the basis that is canonical in Alice's Hilbert space, diagonal in Bob's Hilbert space and circular in charley's Hilbert space.

Verify that your answers are consistent with the results of the first two parts of section (a) of this problem.

c) Show that $|\Psi_{GHZ}\rangle$ is an eigenstate of the operators

- $\hat{\sigma}_x \otimes \hat{\sigma}_y \otimes \hat{\sigma}_y$,
- $\hat{\sigma}_y \otimes \hat{\sigma}_x \otimes \hat{\sigma}_y$,
- $\hat{\sigma}_y \otimes \hat{\sigma}_y \otimes \hat{\sigma}_x$,
- $\hat{\sigma}_x \otimes \hat{\sigma}_x \otimes \hat{\sigma}_x$

with eigenvalues -1, -1, -1, $+1$, respectively. **Hint:** this part might be easier to solve if you write the Pauli operators in the Dirac notation in terms of their eigenstates and eigenvalues.