University of Calgary
Winter semester 2011

PHYS 443: Quantum Mechanics I

Final examination

April 27, 2011, 12:00-15:00 (3 hours)

Total points: 100. Open books. No communication equipment allowed. You must solve all problems in order to receive full credit. Partial credit will be given. Use the booklet provided to write your solutions.

Problem 1 (10 pts). Write the decomposition of the state

$$|\psi\rangle = \hat{P}\,|\alpha\rangle\,, \tag{1}$$

where $\hat{P}$ is the rescaled momentum operator and $|\alpha\rangle$ is a coherent state with *real* amplitude α,

a) into the position basis;

b) into the Fock basis.

Problem 2 (10 pts). Alice, Bob, and Charley share an entangled state of three photons

$$|\Psi\rangle = (3\,|+-+\rangle + 4\,|-+-\rangle)/5. \tag{2}$$

Alice and Bob measure their photons in the canonical basis. Alice detects horizontal, and Bob vertical polarization.

a) What is the probability of this event?

b) Onto which state will Charley's photon project?

Problem 3 (20 pts). As discussed in class, the potential well of the form $V(x) = -W_0\delta(x)$ has a single bound state with energy $E = -\hbar^2 k_0^2/2m$ where $k_0 = W_0 m/\hbar^2$ and wavefunction $\psi(x) = \sqrt{k_0}e^{-k_0|x|}$ Suppose the particle is initially in this state. At time $t = 0$, the potential well instantly displaces by distance a in the positive x direction. What is the probability to find the particle far away from the well at $t = \infty$?

Problem 4 (30 pts). Consider the state of the harmonic oscillator whose decomposition in the photon number basis has the form

$$|\psi(t=0)\rangle = \alpha\,|0\rangle - \beta\,|2\rangle\,,$$

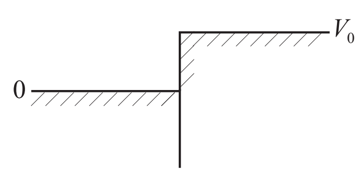
where α and β are real and positive; $\alpha^2 + \beta^2 = 1$.

a) Find the wavefunction of $|\psi(t=0)\rangle$ in the position basis.

b) Find the behavior $|\psi(t)\rangle$ of this state as a function of time in the photon number basis.

c) Find the expectation value and the variance of the energy as a function of time.

d) Find the expectation value and the variance of the position as a function of time.



Problem 5 (30 pts). Find the energies and wavefunctions of all bound states associated with the potential

$$V(x) = V_0\theta(x) - W_0\delta(x),$$

where V_0 and W_0 are positive and $\theta(x)$ is the Heaviside step function (Fig.). Find the conditions for the existence of at least one bound state.

Finding the normalization constants of the wavefunctions is not required.

Some useful information

Wavefunctions of the first three energy eigenstates of the harmonic oscillator:

$$\psi_0(X) = \frac{1}{\pi^{1/4}}e^{-X^2/2};$$

$$\psi_1(X) = \frac{\sqrt{2}}{\pi^{1/4}}Xe^{-X^2/2};$$

$$\psi_2(X) = \frac{1}{\sqrt{2}\pi^{1/4}}(2X^2 - 1)e^{-X^2/2}.$$