PHYS 443

Second Midterm Examination

Solutions

1

$$\begin{cases} |R\rangle = \dfrac{|H\rangle + i|V\rangle}{\sqrt{2}} \\ |L\rangle = \dfrac{|H\rangle - i|V\rangle}{\sqrt{2}} \end{cases} \Rightarrow \quad |H\rangle = \frac{|R\rangle + |L\rangle}{\sqrt{2}} \qquad |V\rangle = \frac{|R\rangle - |L\rangle}{\sqrt{2}\,i}$$

$$|\Phi^{\pm}\rangle = \frac{1}{2\sqrt{2}}\left[(|R\rangle + |L\rangle)(|R\rangle + |L\rangle) \pm \underbrace{\frac{1}{i^2}}_{=-1}(|R\rangle - |L\rangle)\otimes(|R\rangle - |L\rangle)\right]$$

$$|\Phi^{+}\rangle = \frac{1}{\sqrt{2}}\left(|LR\rangle + |RL\rangle\right)$$

$$|\Phi^{-}\rangle = \frac{1}{\sqrt{2}}\left(|RR\rangle + |LL\rangle\right)$$

2

$$\hat{V} = V(\hat{x}) = \int V_0 e^{-x^2/d^2} |x\rangle\langle x|\, dx$$

a) $$\langle x|V|x'\rangle = \int V_0 e^{-x''^2/d^2} \langle x|x''\rangle\langle x''|x'\rangle\, dx''$$

$$= \int V_0 e^{-x''^2/d^2} \delta(x - x'')\,\delta(x'' - x')\, dx'' = V_0 e^{-x^2/d^2}\delta(x - x')$$

b) $$\langle p|V|p'\rangle = \iint \langle p|x\rangle\langle x|V|x'\rangle\langle p'|x'\rangle^{*}\, dx\, dx'$$

$$= \frac{1}{2\pi\hbar}\iint e^{i\frac{-px + p'x'}{\hbar}} V_0 e^{-x^2/d^2}\delta(x - x')\, dx\, dx'$$

$$= \frac{V_0}{2\pi\hbar}\int e^{\frac{ix(p'-p)}{\hbar}} e^{-x^2/d^2}\, dx$$

$$= \frac{V_0}{\sqrt{2\pi}\hbar}\, \mathcal{F}\left[e^{-x^2/d^2}\right]\left(\frac{p'-p}{\hbar}\right) = \frac{V_0}{\sqrt{2\pi}\hbar}\frac{d}{\sqrt{2}} e^{-\frac{(p'-p)^2 d^2}{4\hbar^2}}$$

$\boxed{3}$ From the measurement results in Alice's diagonal basis,

$$|\psi\rangle = a\,|+H\rangle + b\,|-V\rangle =$$

$$= \frac{a}{\sqrt{2}}|HH\rangle + \frac{a}{\sqrt{2}}|VH\rangle + \frac{b}{\sqrt{2}}|HV\rangle - \frac{b}{\sqrt{2}}|VV\rangle$$

$$= |H\rangle\left(\frac{a}{\sqrt{2}}|H\rangle + \frac{b}{\sqrt{2}}|V\rangle\right) + |V\rangle\left(\frac{a}{\sqrt{2}}|H\rangle - \frac{b}{\sqrt{2}}|V\rangle\right)$$

From measurement in Alice's canonical basis, we have

$$\frac{a}{\sqrt{2}}|H\rangle + \frac{b}{\sqrt{2}}|V\rangle = |L\rangle$$

$$\frac{a}{\sqrt{2}}|H\rangle - \frac{b}{\sqrt{2}}|V\rangle = |R\rangle$$

$$\Rightarrow a = 1, \quad b = -i$$

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|+H\rangle - i\,|-V\rangle\right)$$

(This answer can also be written as

$$|\psi\rangle = \frac{1}{2}\left(|HH\rangle + |VH\rangle - i|HV\rangle + i|VV\rangle\right)$$

$$= \frac{1}{\sqrt{2}}\left(|HL\rangle + |VR\rangle\right))$$