PHYS 443: Quantum Mechanics I

First midterm examination

February 15, 2007

Open books. Attempt all questions. Partial credit will be given.

<u>Problem 1.</u> An atom has three energy levels, $E_1 = 0$, $E_2 = \hbar\omega$, $E_3 = 4\hbar\omega$. This corresponds to a Hamiltonian

$$\hat{H} = E_1 |E_1\rangle\langle E_1| + E_2 |E_2\rangle\langle E_2| + E_3 |E_3\rangle\langle E_3|. \tag{1}$$

a) (10 pts) The atom is in the state $|\psi_0\rangle = (|E\rangle_1 - 2i |E_2\rangle + 3 |E_3\rangle)/\sqrt{14}$. What is the probability to detect the atom in the second energy eigenstate?

b) (10 pts) What is the expectation value of the energy measured in the state $|\psi_0\rangle$?

c) (20 pts) What is the probability that the atom, initially prepared in the state $|\psi_0\rangle$, will remain in this state after it has evolved under Hamiltonian (1) for the time $t = \pi/(2\omega)$?

<u>Problem 2.</u> (30 pts) Find $e^{i\frac{\pi}{4}\left(3|H\rangle\langle H| + \sqrt{3}i|H\rangle\langle V| - \sqrt{3}i|V\rangle\langle H| + |V\rangle\langle V|\right)}$.

<u>Problem 3.</u> Consider an apparatus for measuring the polarization of a photon shown in the figure below. The display shows "1" when detector 1 fires, and "2" when detector 2 fires.

a) (15 pts) What are the eigenstates and eigenvalues of the observable measured with this apparatus?

b) (15 pts) Write the matrix of this observable in the canonical basis.