University of Calgary
Winter semester 2007

PHYS 443: Quantum Mechanics I

Homework assignment 2

Due February 6, 2007

Problem 2.1. As discussed in class, quantum cryptography becomes insecure when the measurement error rate due to dark counts of Bob's single photon detectors exceeds ∼25% of all detection events. Assuming that Alice has a perfect single photon source, estimate the maximum possible secure communication distance and the bit transfer rate at this distance given the following parameters:

- photon loss in the fiber communication line: 5%/km;
- emission rate of Alice's source: 10^6 photons per second;
- quantum efficiency of the photon detectors (i.e. the probability that the detector will "click" when hit by a single photon): 10%;
- dark count rate of the photon detectors: 10^{-5} per pulse.

Problem 2.2. Find the matrices of the linear operators from Ex. 1.33 in the lecture notes.

Problem 2.3. A "one-sixth wave plate" with its optical axis oriented vertically leaves the horizontally polarized light unaffected, but applies a phase delay of $\pi/3$ to vertically polarized light. For the operator $\hat{A}$ associated with this optical element:

a) write explicitly how states $|H\rangle$ and $|V\rangle$ are mapped by $\hat{A}$;

b) write the matrix of $\hat{A}$ in the canonical basis;

c) using only the definition of the linear operator determine how $\hat{A}$ acts upon the diagonal polarization states;

d) using the previous result, find the matrix of $\hat{A}$ in the diagonal polarization basis;

e) find the matrix of $\hat{A}$ in the canonical basis from its matrix in the diagonal basis using the method described in Note 1.20 from the lecture notes. Is your result consistent with that of part (b)?

f) find the traces of the matrices of $\hat{A}$ in the canonical and diagonal bases. Are they identical?

g) express $\hat{A}$ in the Dirac notation in terms of outer products of states $|H\rangle$ and $|V\rangle$;

h) is $\hat{A}$ Hermitean? If not, what is its adjoint?

Problem 2.4. Consider an apparatus for measuring the photon polarization that has the following properties:

- whenever it is fed with a $60°$ polarized photon, it displays "1"
- whenever it is fed with a $-30°$ polarized photon, it displays "-1"

a) Find the eigenvalues and the eigenstates of the operator $\hat{A}$ associated with the observable measured by this apparatus (**Hint:** you need not solve any equations);

b) find the matrices of $\hat{A}$ in its eigenbasis and in the $\{|H\rangle, |V\rangle\}$ basis;

c) find the probability of each measurement outcome for a $30°$ polarized input photon;

d) find the expectation value of this measurement using (i) Definition 1.26 from the lecture notes and the result of part (c) and (ii) Eq. (1.33) from the lecture notes. Verify that these results are identical.