University of Calgary
Winter semester 2007

PHYS 443: Quantum Mechanics I

Homework assignment 3

Due February 13, 2007

Problem 3.1. In a three-dimensional Hilbert space, three operators, in an orthonormal basis $\{|v_1\rangle, |v_2\rangle, |v_3\rangle\}$ have the following matrices:

$$\hat{L}_x \leftrightarrow \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \hat{L}_y \leftrightarrow \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \hat{L}_z \leftrightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

a) Show that these operators are Hermitian (i.e. they can be interpreted as physical observables).

b) Are these operators unitary?

c) Find the eigenvalues and eigenstates of $\hat{L}_x$, $\hat{L}_y$, and $\hat{L}_z$.

d) Find the commutation relations of these observables.

e) The observable $\hat{L}_y$ is measured in the state $|\psi\rangle = (3|v_1\rangle + 4i|v_2\rangle)/5$. What results can be obtained and with which probabilities?

f) Find the expectation values and uncertainties of the measurements of $\hat{L}_x$ and $\hat{L}_y$ in the state $|\psi\rangle$.

g) Verify that the uncertainty principle holds for the measurements in part (f).

h) The system initially ($t = 0$) in state $|v_1\rangle$ experiences quantum evolution with the Hamiltonian $\hat{H} = \hbar\omega\hat{L}_x$. Find the state of the system at an arbitrary time t. What is the probability that the system will remain in its initial state at the moment $\omega t = \pi/2$? $\omega t = \pi$? $\omega t = 2\pi$?