University of Calgary
Winter semester 2007

PHYS 443: Quantum Mechanics I

Homework assignment 6

Due April 3, 2007

Problem 6.1. A particle is in the ground state of an infinite potential box of length a. Suddenly the box expands (symmetrically) to twice its size. What is the probability of finding the particle in the ground state of the new potential?

Problem 6.2. Consider the state $\psi(x) = \begin{cases} Ax \text{ for } |x|<a/2 \\ 0 \text{ for } |x|\geq a/2 \end{cases}$ ($A = 2\sqrt{3}/a^{3/2}$ being the norm) in an infinite potential box $V(x) = \begin{cases} +\infty \text{ for } |x|>a/2 \\ 0 \text{ for } |x|\leq a/2 \end{cases}$. Find the energy spectrum of this state, i.e. the probabilities $\mathrm{pr}(E_n)$ to measure each energy eigenvalue. Show that these probabilities sum up to 1. (**Hint:** $\sum 1/n^2 = \pi^2/6$.)

Problem 6.3. Find the equation for the energies associated with even and odd solutions of the time-independent Schrödinger equation for an infinite potential box with a delta-function in the middle:

$$V(x) = V_0\delta(x) + \begin{cases} +\infty \text{ for } |x| > a/2 \\ 0 \text{ for } |x| \leq a/2 \end{cases}$$

Find (numerically if necessary) the lowest even and odd energy solutions.

Problem 6.4. Consider a wavefunction that at the moment $t = 0$ has the form of a Gaussian wavepacket

$$\psi(x) = \frac{1}{(\pi d^2)^{1/4}} e^{i\frac{p_0 x}{\hbar}} e^{-\frac{x^2}{2d^2}}. \tag{1}$$

As showed in class, such a state has average momentum p_0. Neglecting the spreading (when can you do this?), show that the time evolution of this wavepacket is simply (up to a phase factor) displacement by a distance $p_0 t/m$.

Problem 6.5. Recalling that $\mathrm{pr}(x) = \psi(x)\psi^*(x)$, derive the continuity equation:

$$\frac{d\mathrm{pr}(x)}{dt} = -\frac{dj}{dx}, \tag{2}$$

where

$$j = -i\frac{\hbar}{2m}\left(\psi^*\frac{d\psi}{dx} - \psi\frac{d\psi^*}{dx}\right) \tag{3}$$

is the probability density current.

Problem 6.6. Find the transcendental equation defining the energy eigenvalues and the bound stationary states of the potential

$$V(x) = \begin{cases} +\infty \text{ for } x \leq 0; \\ 0 \text{ for } 0 < x \leq a; \\ V_0 \text{ for } x > a. \end{cases}$$

Verify that in the limit $V_0 \to \infty$, the energy eigenvalues become equal to those obtained in class for an infinite potential box.