University of Calgary
Winter semester 2007

PHYS 443: Quantum Mechanics I

Homework assignment 7

Due April 12, 2007

Problem 7.1. For an arbitrary $|n\rangle$, calculate $\langle X\rangle$, $\langle \Delta X^2\rangle$, $\langle P\rangle$, $\langle \Delta P^2\rangle$ and verify the uncertainty principle. **Hint:** do *not* use wavefunctions.

Problem 7.2. A finite potential barrier of very large height V_0 and very short length L can be approvimated as a delta-function potential $V(x) = W\delta(x)$, where $W = V_0 L$. Find the reflection and transmission coefficients of a de Broglie wave with wavevector k on such a barrier.

Problem 7.3.

a) Find the matrices of $\hat{L}_x$, $\hat{L}_y$, $\hat{L}_z$, $\hat{L}_\pm$, and $\hat{L}^2$ explicitly for $l = 1$.

b) Verify that these matrices obey $\hat{L}_x^2 + \hat{L}_y^2 + \hat{L}_z^2 = \hat{L}^2$.

c) For these matrices, determine the commutators $[\hat{L}_i, \hat{L}_j]$, $[\hat{L}_z, \hat{L}_\pm]$ and $[\hat{L}_+, \hat{L}_-]$ and verify that they are consistent with the general commutation relations for the angular momentum derived in class.

Problem 7.4. Spin-1 particles of mass m, charge e and Landé factor g are prepared with the spin projection onto the x-axis $m = -1$. At $t = 0$, a magnetic field B in the z-direction is turned on. After time t_0 has elapsed, the field is turned off and the electrons are subjected to the Stern-Gerlach measurement with the magnetic field oriented along the x axis. What are the probabilities of all possible measurement outcomes? **Hint:** use the result of homework problem 3.1.